What is a Gros Topos?

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Abstract. F.W. Lawvere [1] has stressed the importance, in studying cohesion axiomatically, of the 'Nullstellensatz' condition relating a topos $\mathcal{E}$ to a base topos $\mathcal{S}$, which turns out [2] to be equivalent to the existence of a geometric morphism $p\colon \mathcal{E} \to \mathcal{S}$ which is locally connected, hyperconnected and local. Important examples include the 'gros topos' of locally connected spaces, and the similar smooth and algebraic 'gros toposes'. But these examples also have the property, stressed in [3], that (at least for suitable objects X of $\mathcal{E}$) one has a similar relationship between the slice $\mathcal{E}/X$ and a 'petit topos' whose objects may be viewed as sheaves on X. If one demands such a relationship for all X, then one has a class $\mathcal{D}$ of 'discrete maps' in $\mathcal{E}$ satisfying axioms closely related (but apparently not equivalent) to those of Joyal and Moerdijk [4] for a 'class of étale maps'. We discuss the axiomatization of this notion, and consider the extent to which the axioms are satisfied in the leading examples.

References.

[1] F.W. Lawvere, Axiomatic cohesion, *Theory Appl. Categ.* 19 (2007), 41–49.

[2] P.T. Johnstone, Remarks on punctual local connectedness, *Theory Appl. Categ.* 25 (2011), 51–63.

[3] P.T. Johnstone and I. Moerdijk, Local maps of toposes, *Proc. London Math. Soc.* (3) 58 (1989), 281–305.

[4] A. Joyal and I. Moerdijk, A completeness theorem for open maps, *Ann. Pure Appl. Logic* 70 (1994), 51–86.

* Inspired by work of Nick Duncan.