Graphically Factoring the Tannaka Construction

Micah Blake McCurdy

Abstract.

Given an algebraic gadget—a monoid, say, or a weak bialgebra, or a braided Hopf algebra—in a monoidal category V, we can consider its category of modules or comodules; these will each come equipped with a so-called "fibre functor" back to V. If V is well-behaved, the functor taking algebraic gadgets to fibre functors will have a left adjoint, called the Tannaka construction. Using our "strings and stripes" graphical notation for monoidal functors, we give a lucid treatment of the Tannaka construction; in particular, it becomes easy to see that the Tannaka construction applied to separable Frobenius monoidal functors gives weak bialgebras. Since, in favourable cases, a Frobenius monoidal functor can be thought of as a Frobenius algebra in a suitably constructed functor category [1], we can give a surprising result showing that the Tannaka construction is essentially the same as a hitherto-unrelated construction [2] of a weak bialgebra from a separable Frobenius algebra.

A gentle talk, with lots of graphical exposition.

References

[1] Egger, Jeff, *Star-Autonomous Functor Categories*, Theory and Applications of Categories, Vol. 20, 2008, No. 11, pp 307-333.

[2] Pastro, Craig, and Ross Street, *Weak Hopf monoids in Braided Monoidal Categories*, Algebra Number Theory v3 (2009), no. 2, pp 149207.