Generalized Tannaka Duality

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Abstract. A classical result of Tannaka duality asserts that for any field k, a rigid k-linear abelian tensor category equipped with a faithful exact k-linear tensor functor to vector spaces is equivalent to the category of finite dimensional comodules of some Hopf algebra. Ross Street observed that there is an adjunction between rigid categories equipped with a strong symmetric monoidal functor to a symmetric monoidal category $\mathcal{V}$ and Hopf algebras in $\mathcal{V}$. The above result characterizes the objects for which the unit is an equivalence in the case where $\mathcal{V}$ is the category of vector spaces.

We give a new construction of the Tannakian adjunction which mimics the construction of the semantics-structure adjunction in the "formal theory of monads". Instead of Eilenberg-Moore objects we consider *Tannaka-Krein* objects, which are universal among coactions on left adjoint 1-cells in a 2-category. For example, any coalgebra C can be thought of as a comonad in the category of k-linear profunctors, and the category of finite dimensional comodules of C is a Tannaka-Krein object for C in the 2-category of profunctors. More generally, we can show that any comonad in the 2-category of $\mathcal{V}$-profunctors has a Tannaka-Krein object. Using this new construction of the Tannakian adjunction we find a list of conditions which ensure that the unit is an equivalence for an arbitrary monoidal base category $\mathcal{V}$.