Exponential objects as Eilenberg-Moore algebras

Gavin J. Seal *

Abstract.

Exponentiable objects in the category Top of topological spaces are known to be topological spaces whose set of continuous maps into the Sierpinski space 2 forms a continuous lattice, that is, those spaces X for which $\mathsf{Top}(X, 2)$ is an algebra relatively to the filter monad $\mathbb{F}$ on Set (see [Isb] for more details).

Since topological spaces are precisely monoids in the Kleisli category $\mathsf{Set}_{\mathbb{F}}$ (see [Gäh]), that is, $\mathsf{Top} \cong \mathsf{Mon}(\mathsf{Set}_{\mathbb{F}})$, the mentioned exponentiability result leads to a characterization of exponentiable objects in categories $\mathsf{Mon}(\mathsf{Set}_{\mathbb{T}})$ of monoids for suitable "powerset-enriched" monads $\mathbb{T}$. The cartesian structure of Top gets replaced by a monoidal structure induced by the monad (following [Koc]), and exponentiable "Kleisli monoids" can be identified as those monoids X for which certain hom-sets $\mathsf{Mon}(\mathsf{Set}_{\mathbb{T}})(X, V)$ are $\mathbb{T}$-algebras.

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