Generalising Connected Components

João J. Xarez

Abstract.

The present work is a generalisation of a previous one by the same author, allowing now to join *geometrical* examples to the known *algebraic* and *topological* examples, in a unified setting. Consider any full reflection $H \vdash I : \mathcal{C} \to \mathcal{M}$, with unit $\eta : 1_{\mathcal{C}} \to HI$, such that $\mathcal{C}$ has pullbacks. Suppose there is a functor $U : \mathcal{C} \to \mathcal{S}$, and a prefactorisation system $(\mathcal{E}, \mathcal{M})$ on $\mathcal{S}$, such that U preserves pullbacks, reflects isomorphisms, and $U(\eta_C)$ is in the largest subclass of $\mathcal{E}$ which is closed under pullbacks in $\mathcal{S}$, $C \in \mathcal{C}$. If a certain lemma also holds for a set $\mathcal{T}$ of objects in the full subcategory $\mathcal{M}$, then it is true that: the reflection $H \vdash I$ is semi-left-exact if and only if its *connected components* are "connected"; it has stable units if and only if certain pullbacks of *connected components* are "connected". The meaning of "connected" is the usual in Galois categorical theory.

References

1. Carboni, A., Janelidze, G., Kelly, G. M., Paré, R., *On localization and stabilization for factorization systems*, App. Cat. Struct. 5 (1997) 1–58.

2. Cassidy, C., Hébert, M., Kelly, G. M., *Reflective subcategories, localizations and factorization systems*, J. Austral. Math. Soc. **38A** (1985) 287–329.

3. Janelidze, G., Laan, V., Márki, L., *Limit preservation properties of the greatest semilattice image functor*, Internat. J. Algebra Comput. 5 (2008) 853–867.

4. Tamura, T., *Attainability of systems of identities on semigroups*, J. Algebra 3 (1966) 261–276.