Monads of measures

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Several algebraic structures allow more or less general kinds of linear combinations. For example, linear spaces allow arbitrary linear combinations, ‘cones’ allow only positive linear combinations, ‘convex spaces’ only convex linear combinations etc. These can been seen as the algebras of some monad $\mathbf{M}$ consisting of the formal linear combinations of the considered type. The Kleisli category of $\mathbf{M}$ describes the corresponding matrices, e.g. arbitrary matrices, positive matrices, stochastic matrices etc.

The idea of my talk is simple: *replace linear combinations by integrals.* For instance, the convex linear combination is the discrete special case of the barycenter of a probability measure. In order to carry out this idea, we replace the formal linear combinations defining the monad $\mathbf{M}$ by measures. However, this somewhat naive idea does not work for general complex measures, the analogue of arbitrary linear combinations; some positiveness or boundedness restrictions are needed.