Operads in higher dimensional category theory

Michael A. Batanin

The idea of operad came from topology and roughly speaking it represents a direction of universal algebra studying a special sort of algebraic theory. However, the significance of this notion goes far beyond just another example of algebraic theory. The point is that operads allow one to handle the algebra up to all higher homotopies. This means that the algebraic laws are not fulfilled strictly but only up to homotopy and this homotopy satisfies other laws also up to homotopy and so on. These ideas have now found large applications not just in topology but also in homological algebra, algebraic geometry and mathematical physics.

The main idea of weak categories is very similar to the idea of homotopical algebra. Instead of the equations between different operations available in a weak category we have now a cell of the next dimension. This cell must satisfy some further equations but also up to a higher cell and so on.

There is no surprise that operads eventually came to higher dimensional category theory. But this also implied further development of the theory of operads. My talk is an overview of a recent development of this theory and the new perspectives this development opens up in higher dimensional category theory and related fields.