# On the homotopy and homology of simplicial complexes

Marco Grandis

A simplicial complex is a set equipped with a down-closed family of distinguished finite subsets. Such objects, usually viewed as codifying triangulated topological spaces, are used here directly, to describe "spaces" whose geometrical realisation is huge and of scarce help. Simplicial complexes form a cartesian closed category, on which structure we build an intrinsic homotopy theory, well related with the – existing – intrinsic homology theory and whose homotopy groups agree with the ones of the geometrical realisation.

Applications are aimed at image analysis in metric spaces, in connection with digital topology and mathematical morphology. A metric space has a structure of simplicial complex at each (positive) resolution; the resulting homotopy and homology groups of degree n detect singularities which can be captured by an $n$-dimensional grid, with edges bound by our resolution; this works equally well for continuous or discrete regions of euclidean spaces, as produced by a scanner. Computation is based on direct, intrinsic methods: van Kampen, Mayer-Vietoris and a study of deformation retracts for such regions.