Change of base for categories enriched in a bicategory

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We describe a new tricategory (essentially a 3-category) $Base$ whose objects are the bicategories. A morphism $F : V \to W$ in $Base$ consists of a span

$$ob(V) \xleftarrow[p]{} S \xrightarrow[q]{} ob(W)$$

of sets, together with functors $F(s,t) : V(ps, pt) \longrightarrow W(qs, qt)$ for each pair s, t in S, along with the data and axioms expressing that F is "like a lax functor". The 2-cells and 3-cells are then what one would expect. Now the representable 3-functor $Base(1, -) : Base \longrightarrow 2\text{-}Cat$ sends V to $V\text{-}Cat$, providing an efficient "change of base" notion for categories enriched in bicategories. In particular, the adjoint pairs in $Base$, which are easy to describe, give adjunctions between $V\text{-}Cat$ and $W\text{-}Cat$. It is further fruitful to see a morphism $F : V \longrightarrow W$ in $Base$ as a "category enriched from V to W"; and for a suitably-complete W this can also be exhibited as a $[V, W]$-category, where $[V, W]$ is a new bicategory constructed by an extension of Day's convolution process.

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