The Chu construction in the context of linear bicategories

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Given a closed (with respect to 1-cell composition) bicategory X with finitely complete hom-categories, at CT97 we used the endo-1-cells of X as the objects of a new closed bicategory that is $*$-autonomous in a suitable sense (here called "cyclicly $*$-autonomous") and hence is a special case of a closed linear bicategory (cf., Robert A. Seely's talk). This bicategorical version of the Chu-construction may also be applied to either open or closed finite chains of composable 1-cells in X of fixed length to produce new closed bicategories. Even chains indexed by the natural numbers $\mathbf{N}$ or by the integers $\mathbf{Z}$ can be used. Chains of length 0 correspond to objects of X. An important difference between using open respectively $\mathbf{N}$-indexed chains and closed respectively $\mathbf{Z}$-indexed chains as objects is that in the latter case the 1-cells are subject to fewer constaints. This results in bicategories that are even cyclicly $*$-autonomous.

Finite closed chains can be "unrolled" into $\mathbf{Z}$-indexed chains, thus providing embeddings of the corresponding bicategories preserving the closed structure. For finite open chains the situation is more complicated. Unless X has dualizing 1-cells (or equivalently, carries a closed linear structure), we cannot extend "short" open chains in such a way that the resulting embedding preserves the closed structure.