A characterization of the left exact categories whose exact completion is a topos

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Many categories of interest arise as exact completions of a left exact category. For example, for every small left exact category $\mathbf{C}$, the presheaf topos $\mathbf{Sets}^{\mathbf{C}^{\mathbf{op}}}$ is an exact completion [1]. Realizability toposes are also examples [4]. More recently, in computer science there has been a lot of interest in the exact completion of the category of topological spaces.

Although a simple construction of the exact completion of a left exact category was given in [2], the resulting category will be usually more difficult to work with directly than the category giving rise to it. So it is interesting to be able to deduce important properties of the former in terms of easily checkable properties of the latter.

For example, a characterization of those left exact categories whose exact completions are (locally) cartesian closed has been given in [3]. This was used to prove that the exact completion of the category of topological spaces is locally cartesian closed.

However, it remained an open question to find a simple characterization of those left-exact categories whose exact completions are toposes. In this talk we provide such a characterization.

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[*]Joint work with Alex Simpson.