Kuratowski Convergence in Approach Spaces

Meryem Ateş*

Ankara University, Faculty of Science, Department of Mathematics, Ankara, TURKEY
mbiten@ankara.edu.tr

In a topological space (X, τ), Kuratowski convergence of hypernets is defined by Beer[1] and in a convergence space (X, ξ), Kuratowski convergence of hyperfilters is defined by Dolecki and Mynard[4]. This definitions lead us to investigate analogue definitions for hypernets and hyperfilters in the setting of approach spaces defined by Lowen[5].

In an approach space (X, δ), for a hypernet $\mathcal{A}$, $x \in X$ and $\varepsilon \in [0, \infty]$, we define the set of the limit points and the cluster points of a hypernet $\mathcal{A}$ in the sense of Kuratowski with $Li_{\delta_\varepsilon}(\mathcal{A})$ and $Ls_{\delta_\varepsilon}(\mathcal{A})$, respectively. And we defined Kuratowski convergence of a hypernet in an approach space as follows:

"$\mathcal{A}$ is said to be $\varepsilon-$Kuratowski convergent to A if $Li_{\delta\varepsilon}(\mathcal{A}) = Ls_{\delta\varepsilon}(\mathcal{A}) = A$."

In a Hausdorff approach space (X, δ), for a hyperfilter $\mathfrak{F}$, $x \in X$ and $\varepsilon \in [0, \infty]$, we define the set of the limit points and the cluster points of a hyperfilter $\mathfrak{F}$ in the sense of Kuratowski and denote these sets with $Li_{\delta_\varepsilon}(\mathfrak{F})$ and $Ls_{\delta_\varepsilon}(\mathfrak{F})$, respectively. And we defined Kuratowski convergence of a hyperfilter in an approach space as follows:

"$\mathfrak{F}$ is said to be $\varepsilon-$Kuratowski convergent to A if $A \in Li_{\delta_\varepsilon}\mathfrak{F} \cap Ls_{\delta_\varepsilon}\mathfrak{F}$."

Then we investigate the relation between $-$ Kuratowski convergence of the hypernet and hyperfilter generated by this hypernet. Also we obtain relations with these new notion of convergence and Fell approach structure defined by Ateş and Sagıroglu in [3].

References

[1] G. Beer, Topologies on Closed Convex Sets, Kluwer Academic Publishers, 1993.

[2] K. Kuratowski, Topology, Vol I, Academic Press, New York, 1966.

[3] M. Ates, S. Sagiroglu, The Fell Approach Structure, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat. (2023)

[4] S. Dolecki, F. Mynard, 2016 *Convergence Foundations of Topology*. World Scientific.

[5] R. Lowen, Approach Spaces: the Missing Link in the Topology Uniformity Metric Triad, Oxford Mathematical Monographs. Oxford University Press, New York, United Stated Springer, 1997.

[6] R. Lowen, Index Analysis, Approach Theory at Work, Springer, 2015.

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