On Equivalence of Asymptotic Average Shadowing and Vague Specification Properties for Topological Dynamical Systems

Melih Emin Can*

address m.emincan930gmail.com

A topological dynamical system is a pair (X,T) consisting of a compact metric space X and a continuous self-map T of X. Recently Downarowicz and Weiss [4] asked what is the relation between the vague specification property (introduced by Teturo Kamae in [2]) and the weak specification property (a variant of the specification property introduced by Bowen in [1]). We prove that the vague specification property is equivalent to the asymptotic average shadowing property, a variant of the classical shadowing property introduced by Gu in [3]. Since the weak specification property implies the asymptotic average shadowing property but the converse is not true, we have a complete answer to Downarowicz and Weiss' question. Additionally, we show that the examples of proximal and minimal shift spaces provided in [5] have the vague specification property.

References

- R. Bowen (1971), Periodic points and measures for Axiom A diffeomorphisms, Transactions of the American Mathematical Society.
- T. Kamae (1975), Normal numbers and ergodic theory, Proceedings of the Third Japan-USSR Symposium on Probability Theory.
- [3] R. Gu (2007), *The asymptotic average shadowing property and transitivity*, Nonlinear Analysis. Theory, Methods & Applications. An International Multidisciplinary Journal
- [4] T. Downarowicz and B. Weiss (2024), *Lifting generic points*, Ergodic Theory and Dynamical Systems.
- [5] M. Can, J. Konieczny, M. Kupsa, and D. Kwietniak (2023) Minimal and proximal examples of *ā*-stable and *ā*-approachable shift spaces, [arXiv:2308.13967], Ergodic Theory and Dynamical Systems (to appear).