Renormalization in Lorenz maps – Completely invariant sets and periodic orbits

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Lorenz maps are piecewise monotone interval maps with a single discontinuity. Such maps appear as Poincaré maps in geometric models of well known Lorenz attractor, but they also have important connections with number theory, fractal geometry and neuronal dynamics. The main objects considered in this talk will be renormalizations of expanding Lorenz maps, i.e. certain return maps of an original Lorenz map to smaller intervals around the discontinuity.

In 2011, Yiming Ding in his paper [1] claimed that there is a one-to-one correspondence between the renormalizations and proper completely invariant closed sets of an expanding Lorenz map. However, in this talk we will present examples showing that the relation between these objects is more delicate. Namely, it turns out that a composition of so-called trivial renormalizations may be a renormalization, which do not have a corresponding proper completely invariant set. Based on this observation we provide complementary information to the statements in [1], which results in a better insight into the structure of renormalizations in Lorenz maps. The talk will be based on a joint work with Piotr Oprocha [2].

References

[1] Y. Ding, *Renormalization and α-limit set for expanding Lorenz maps*, Discrete Contin. Dyn. Syst. **29** (2011), 979–999.

[2] Ł. Cholewa, P. Oprocha, *Renormalization in Lorenz maps – Completely invariant sets and periodic orbits*, preprint, arXiv:2104.00110.

*This is joint work with Piotr Oprocha (AGH University of Science and Technology).