A homological bound on entropy in arbitrary compact spaces

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A result of Manning states that for a compact manifold X and a continuous map $f\colon X \to X$ the topological entropy of f is bounded below by the logarithm of the spectral radius of the map induced by f in the first homology group $H_1(X)$. We generalize this result to arbitrary compact spaces X in terms of Čech cohomology $\check{H}^1(X)$. The essential tool is a notion of integration of Alexander-Spanier cocycles over Čech cycles. Most of the discussion is carried out "at scale $\mathcal{U}$", for an open covering $\mathcal{U}$. This is used to keep track of the "homological length" of the iterates of a cycle, which in turn leads to a lower bound on the number of elements in the coverings that appear in the definition of the entropy.

References

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[2] A. Manning: *Topological entropy and the first homology group,* Lecture Notes in Math., Vol. 468 Springer-Verlag, Berlin-New York, 1975, 185–190.

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