Adherence determined convergences

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Given a *convergence* ξ on X and a family $\mathcal{A}$ of subsets of X, we define the *adherence* $adh_\xi \mathcal{A} := \bigcup_{\mathcal{A}\#\mathcal{F}} \lim_\xi \mathcal{H}$, where $\mathcal{A}\#\mathcal{F}$ means that $A \cap F \neq \emptyset$ for every $A \in \mathcal{A}$ and $F \in \mathcal{F}$. It is clear that $\lim_\xi \mathcal{F} \subset adh_\xi \mathcal{F}$ for every filter $\mathcal{F}$ on X. Let H be a class of filters. A convergence ξ is called H-*adherence determined* if, for every filter $\mathcal{F}$ on X,

$$\lim_\xi \mathcal{F} \supset \lim_{A_H \xi} \mathcal{F} := \bigcap_{H \ni \mathcal{H} \# \mathcal{F}} adh_\xi \mathcal{H}. \tag{1}$$

In particular, if $H = F$ (the class of all filters), then (1) holds if and only if ξ is a *pseudotopology*; if $H = F_0$ (the class of all principal filters), then (1) holds if and only if ξ is a *pretopology*. Similarly, *paratopologies* correspond to the class of countably based filters, and *hypotopologies* to countably complete filters.

If a class H is *initial* ($\mathcal{H} \in H$ then $f^-[\mathcal{H}] \in H$, where $f^-[\mathcal{H}] := \{f^-(H) : H \in \mathcal{H}\}$), then A_H defined in (1) is a concrete reflector (defined on objects ξ of the (topological) category of convergences with continuous maps as morphisms).

Adherence-determined subcategories of convergences are fundamental for a unification of various aspects of convergence theory. Indeed, it is long known that *compactness, countable compactness, Lindelöf property*, and *finite compactness* can be characterized in terms of the reflectors listed above and so-called characteristic convergences. It is also known that classical variants of quotient maps correspond to various subcategories of adherence-determined convergencs. Similar representations hold for variants of perfect maps.

Is is long known that the categories of topologies and pretopologies are simple. More than three decades ago it was proved by Eva and Robert Lowen that neither the category of convergences nor that of pseudotopologies is simple. Recently Jerzy Wojciechowski showed that the category of paratopologies is simple and that of hypotopologies is not. Finally, all the before-mentioned results follow from a complete characterization of the categories of adherence-determined convergences which are simple by Frédéric Mynard and Jerzy Wojciechowski.