On some applications of the Steinhaus chessboard theorem

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The Steinhaus chessboard theorem states: *Let some segments of the chessboard be mined. Assume that the king cannot go across the chessboard from the left edge to the right one without meeting a mined square. Then the rook can go from upper edge to the lower one moving exclusively on mined segments.*

The theorem was generalized in two versions by Tkacz and Turzański [1] to the n-dimensional chessboard. The simpler one states: *Let each segment of the n-dimensional chessboard be colored with one of the n colors. Then there exist pair of opposite faces of the chessboard and connected chain of monochromatic segments which connects these faces.* They also showed that the Poincaré–Miranda theorem and its parametric extension are consequences of their result.

In this talk we shall consider a similar problem to the n-dimensional Steinhaus chessboard theorem with some restrictions on the arrangement of colors. The obtained result is related with that of Tkacz and Turzański and it is proven using the n-dimensional Steinhaus chessboard theorem and the concept of clustered chromatic number [2] which comes from the graph theory.

Its consequence is some Poincaré–Miranda type theorem which, to the best of our knowledge, is unknown. It turns out that this new theorem can be used to prove the n-dimensional Steinhaus chessboard theorem relatively easily, which means that these results are equivalent. Independently, we can apply the new theorem to prove the Brouwer fixed point theorem which is equivalent to the Poincaré–Miranda theorem.

References

[1] P. Tkacz and M. Turzański. An n-dimensional version of Steinhaus' chessboard theorem. *Topology and its Applications*, 155(4):354–361, 2008.

[2] D. R. Wood. Defective and Clustered Graph Colouring. *The Electronic Journal of Combinatorics*, 2012.

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