A study of a new dimension for finite lattices

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The branch of Dimension Theory of topological spaces has been studied extensively, defining some of the most known dimensions; the covering dimension, the small inductive dimension and the large inductive dimension (see, for example, [12, 21, 19, 20, 5]). Also, the dimension Dind for topological spaces, defined by A. V. Arhangelskii (see [8, 11, 18, 13]) has attracted a special interest of such research topics.

However, various topological notions of dimensions, like the covering dimension, the small inductive dimension and the large inductive dimension, have been studied in the "environment" of partially ordered sets, lattices and frames (see [1, 3, 4, 6, 7, 9, 10, 17, 2, 22, 16, 14, 15]).

Inspired by the dimension Dind of A. V. Arhangelskii, in this talk we insert the meaning of such dimension Dind in the class of finite lattices. We characterize it using the so-called minimal covers and study various of its properties like the sublattice property. We study the behavior of this dimension under the view of linear sum, lexicographic, Cartesian and rectangular product. In addition, we investigate relations between Dind and known dimensions of finite lattices like the small inductive dimension, the large inductive dimension, the covering dimension and the Krull dimension.

The results in this talk have been presented in the research paper titled "A study of a new dimension for finite lattices" which is a joint work of D. Georgiou, Y. Hattori, A. Megaritis and F. Sereti.

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