A small Boolean algebra that is Grothendieck but not Nikodym

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For a Boolean algebra $\mathbb{B}$ we say that it has the Grothendieck property, if every weak*-convergent sequence of Radon measures on the Stone space $\mathrm{St}(\mathbb{B})$ is also weakly convergent. We say that $\mathbb{B}$ has the Nikodym property, if every pointwise convergent sequence of measures on $\mathbb{B}$ is bounded in norm. In 1984 Talagrand showed that under the continuum hypothesis there is a Boolean algebra with the Grothendieck property and without the Nikodym property, but the problem of the existence of such an algebra is still open in ZFC.

We construct a σ-centered notion of forcing that forces the existence of a Boolean algebra of cardinality ω_1, which has the Grothendieck property, but does not have the Nikodym property. In particular, the existence of such an algebra is consistent with any possible value of the continuum.

References

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