Functionally countability vs exponential separability

Rodrigo Hernández-Gutiérrez*

Department of Mathematics, Universidad Autónoma Metropolitana campus Iztapalapa, Mexico city, Mexico rod@xanum.uam.mx

A space X is functionally countable if the image of X under any continuous function is countable. In [3] Tkachuk defines the class of exponentially separable spaces, which includes all spaces X such that every closed subspace of X is functionally countable. In recent joint work with Santi Spadaro ([2]), we continued Tkachuk's quest on finding classes of exponentially separable spaces. Among our main results we have that: (i) several classes of generalized ordered spaces that were known to be functionally countable (as shown in [1] and [5]) are in fact exponentially separable, (ii) there are examples of functionally countable, non-exponentially separable spaces of arbitrarily large extent, (iii) the function space $C_p(X, 2)$ is exponentially separable when X is a Gul'ko compactum (this expands a result from [4]), and (iv) there is an example of a separable and uncountable exponentially separable space in ZFC. In this talk, I will present these results and speak about some open questions.

References

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^{*}This is joint work with Santi Spadaro (University of Palermo).