A topological insight into the polar involution of convex sets

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Let us denote by $\mathcal{K}_0^n$ the family of all closed convex sets in the n-dimensional euclidean space $A \subset \mathbb{R}^n$ containing the origin $0 \in \mathbb{R}^n$. For $A \in \mathcal{K}_0^n$, its polar set is denoted by A°. Namely,

$$A^\circ := \left\{ x \in \mathbb{R}^n : \sup_{a \in A} \langle a, x \rangle \leq 1 \right\},$$

where $\langle \cdot, \cdot \rangle$ stands for the usual inner product on $\mathbb{R}^n$.

In this talk, we will discuss the topological nature of the polar map $A \to A^\circ$ on $(\mathcal{K}_0^n, d_{AW})$, where d_{AW} denotes the Attouch-Wets metric. We will show that $(\mathcal{K}_0^n, d_{AW})$ is homeomorphic to the Hilbert cube $Q = \prod_{i=1}^{\infty}[-1, 1]$ and that the polar map is topologically conjugate with the standard based-free involution $\sigma : Q \to Q$, defined by $\sigma(x) = -x$ for all $x \in Q$. We will also characterize all the inclusion-reversing involutions on $\mathcal{K}_0^n$ which are conjugate σ. In this sense, we will show that the polar map is essentially the only possible decreasing involution with a unique fixed point on $\mathcal{K}_0^n$.

References

[1] L. F. Higueras-Montaño, N- Jonard Pérez, *A topological insight into the polar involution of convex sets*. (To appear in Israel Journal of Mathematics).

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