On spaces ℓ_∞ and c_0 with the pointwise topology embedded into spaces $C_p(X)$

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The linear space of all continuous real-valued functions on a Tychonoff space X with the pointwise topology (induced from the product space R^X) is denoted by $C_p(X)$. We continue a study of sequences spaces c_0 and ℓ_q (for $0 < q \leq \infty$) with the topology induced from R^N (denoted by $(c_0)_p$ and $(\ell_q)_p$, respectively) and their role in the theory of $C_p(X)$ spaces. It is known (Banakh-Kąkol-Śliwa) that $C_p(X)$ contains a complemented copy of $(c_0)_p$ if and only if $C_p(X)$ has the Josefson-Nissenzweig property if and only if $C_p(X)$ admits a linear continuous map onto $(c_0)_p$. On the other hand, one shows that for every infinite Tychonoff space X the space $C_p(X)$ contains an (isomorphic) copy of $(c_0)_p$; if X contains an infinite compact subset, then the copy of $(c_0)_p$ is closed in $C_p(X)$. It follows that $C_p(X)$ contains a copy of $(\ell_q)_p$ for every $0 < q \leq \infty$. Nevertheless, for any infinite compact space X the space $C_p(X)$ contains no closed copy of $(\ell_q)_p$ for $q \in (0, 1] \cup \{\infty\}$ and no complemented copy for $0 < q \leq \infty$. Relation with results of Talagrand, Haydon, Levy and Odell will be also discussed.