Tameness and amorphic complexity of constant length substitution systems

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Amorphic complexity—introduced by Fuhrmann, Gröger, and Jäger [1]—is a relatively new invariant of topological dynamical systems useful in the study of aperiodic order and low complexity dynamics. Tameness is a well-studied notion usually defined in terms of the size of the Ellis semigroup of the system.

In our work we study amorphic complexity and tameness in the class of automatic systems—systems arising from constant length substitutions. We provide a closed formula for the amorphic complexity of any automatic system and show that tameness of such systems can be succinctly characterized through amorphic complexity: A minimal automatic system is tame if and only if its amorphic complexity is zero (in which case the system is finite) or one. This extends and complements earlier works of Fuhrmann and Gröger [2] and Fuhrmann, Kellendonk, and Yassawi [3].

Our proofs use methods from fractal geometry and introduce some new dynamically-defined pseudometrics. These methods seem suitable for study of other systems of S-adic nature including Toeplitz subshifts. Time permitting we will touch on some possible generalisations in these directions.

References

[1] G. Fuhrmann, M. Grger, and T. Jäger. Amorphic complexity. Nonlinearity, 29(2):528–565, 2016.

[2] G. Fuhrmann and M. Gröger. Constant length substitutions, iterated function systems and amorphic complexity. Mathematische Zeitschrift, 295:1385–1404, 2020.

[3] G. Fuhrmann, J. Kellendonk, and R. Yassawi. Tame or wild Toeplitz shifts. Ergodic Theory Dynam. Systems, 44(5):1379–1417, 2024.

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