

Renorming Problem for non-archimedean normed spaces

ALBERT KUBZDELA

Poznan University of Technology, Poland
albert.kubzdela@put.poznan.pl

Let \mathbb{K} be a non-archimedean complete valued field and $(E, \|\cdot\|)$ be a non-archimedean normed space over \mathbb{K} . Let $|\mathbb{K}^\times| := \{|\lambda| : \lambda \in \mathbb{K} \setminus \{0\}\}$ and $\|E^\times\| := \{\|x\| : x \in E \setminus \{0\}\}$. In general $|\mathbb{K}^\times|$ may differ from $\|E^\times\|$. By Renorming Problem we mean the following question: Can one for every non-archimedean normed space E introduce a norm $\|\cdot\|_\bullet$ on E that is equivalent to the given norm, i.e. determines the same topology and has the property $\|E^\times\|_\bullet = |\mathbb{K}^\times|$?

It turned out that a solution strictly depends on the type of valuation.

The first solution has presented by J.P. Serre in 1962 for discretely valued \mathbb{K} . In 1976 A. van Rooij demonstrated the affirmative solution for a densely valued \mathbb{K} and a non-archimedean normed space E for which $\|E^\times\|$ is an union of countably many cosets of $|\mathbb{K}^\times|$.

I would like to present the solution of this Problem when \mathbb{K} is densely valued without any additional assumptions about E .

References

- [1] A. Kubzdela, Serre's Renorming Problem, Bull. Belg. Math. Soc. Simon Stevin 31 (2024), in press
- [2] J. P. Serre, Endomorphismes completement continus des espaces de Banach p-adiques, Institut Des Hautes Etudes Scientifiques, 12 (1962), 70-85.