Embedding the free topological group $F(X^n)$ into $F(X)$

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A very well-known fact says that the free algebraic group on two generators $\mathbb{F}_2$ contains an isomorphic copy of the free algebraic group $\mathbb{F}_\infty$ on a countably infinite set of generators. Consequently, the questions that follow can be viewed as topological versions of this purely algebraic result.

Below $X \times X = X^2$ denotes the square of X, and $X \oplus X$ denotes the free topological sum of two copies of X.

Problem. *Let X be a Tychonoff topological space with $|X| \geq 2$.*

(1) *Is it true that the free topological group $F(X)$ contains a (closed) subgroup topologically isomorphic to $F(X \times X)$?*

(2) *Is it true that $F(X)$ contains a (closed) subgroup topologically isomorphic to $F(X \oplus X)$?*

We show that for every topological space X such that all finite powers of X are pseudocompact, $F(X^n)$ is topologically isomorphic to a closed subgroup of $F(X)$ for each natural n. In particular, all pseudocompact k-spaces enjoy this property [1].

Our results are quite sharp because we present the first example of a Tychonoff countably compact space Z such that $F(Z)$ does not contain an isomorphic copy of the group $F(Z^2)$. In addition, our space Z is separable, and its square Z^2 is not pseudocompact [1].

The second item of Problem above is still open. Analogous questions for the free abelian topological group $A(X)$ are completely resolved in [2].

References

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[2] M. Krupski, A. Leiderman and S. A. Morris, *Embedding of the free abelian topological group $A(X \oplus X)$ into $A(X)$*, Mathematika, **65** (2019), 708–718.

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