Equivariant means and $\mathbb{Z}_2$-ARs

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In the 70's, the following problem was posed by J. Jaworowski.

Jaworowski's problem. Let G be a compact Lie group and X a metrizable G-space that has finitely many G-orbit types. Assume that for each compact subgroup H of G, the H-fixed point set X^H is an AR. Is then X a G-AR?

Although many attempts have been made to answer this question, this problem remains open even in the simplest case when $G = \mathbb{Z}_2$ and the set $X^{\mathbb{Z}_2}$ is a singleton. If X is homeomorphic with the Hilbert cube $Q = \prod_{n=1}^{\infty} [-1, 1]$, this particular case is equivalent to an open problem known as Anderson's conjecture ([1]). Notice that every action of $\mathbb{Z}_2$ on a space X induces an involution $\alpha : X \to X$, given by $\alpha(x) = -1 \cdot x$. Conversely, an involution α on X induces an action of $\mathbb{Z}_2$ on X. We will denote the resulting $\mathbb{Z}_2$-space by (X, α).

A partial positive answer to Anderson's conjecture was found in [2] for the special case when the involution α is decreasing with respect to a certain partial order on Q. Since the order played an important role in this case, it is natural to ask whether it does in a more general setting.

In this talk, we will discuss conditions for a metrizable $\mathbb{Z}_2$-space (X, α) to be a $\mathbb{Z}_2$-AR, provided that X is an AR and there exists a lattice structure $(X, \leq, \wedge, \vee)$ such that α is decreasing with respect to the partial order $\leq$. We will see that the existence of an equivariant, symmetric map $g : X \times X \to X$ satisfying $g(x, x) = x$ for all $x \in X$ is sufficient for X to be a $\mathbb{Z}_2$-AR, and we will talk about when such a function exists. In particular, it exists if the operators $\wedge$ and $\vee$ are continuous and the lattice $(X, \leq, \wedge, \vee)$ is modular. We will also briefly discuss a particular case of Jaworowski's problem for finite groups.

References

[1] S. Antonyan. *Some open problems in equivariant infinite-dimensional topology.* Topology Appl. 311 (2022), 1-12.

[2] L. Higueras-Montaño and N. Jonard. *A topological insight into the polar involution of convex sets.* Accepted in Israel Journal of Mathematics.

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