

# Free distributive and extensive categorical structures

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The study of freely generated categorical structures sits at the intersection of category theory with logic, type theory, algebraic topology, programming language semantics, and other fields. Despite its significance, the existing literature often lacks a thorough treatment of fundamental aspects.

In our work on Grothendieck descent theory [1, 3], presentation of categorical structures [2], automatic differentiation [4], and denotational semantics [5, 6], we have studied freely generated categorical structures and their presentations in various settings.

More recently, motivated by their prominent interest in all these settings, we have studied freely generated distributive and infinitary extensive categories through the lens of two-dimensional monad theory. In particular, in [8, 9], we show how several *free extensive/distributive completions* arise from (pseudo)distributive laws involving the free coproduct completion.

Moreover, we have uncovered aspects on the comparison of *completions* against *free completions*, which, in the setting of two-dimensional monad theory, are encompassed by idempotent (pseudo)monads and lax idempotent ones.

Furthermore, driven by [4, 7], we have studied exponentiability of objects of freely generated categorical structures. We show how connected objects on some of these freely generated categorical structures are exponentiable.

Finally, we also have introduced natural notions of distributivity and extensivity; namely, doubly-infinitary distributive/lexensive categories. We show that the free ones are cartesian closed and we discussed examples; namely, the category of topological spaces is not doubly-infinitary lexensive, while the category of locally connected topological spaces is.

In this talk, we intend to address some of these insights, focusing on very elementary aspects of freely generated distributive and (infinitary l)extensive categories. We also discuss examples, especially focusing on the topological ones. The talk is mostly based on [8, 9] and ongoing work.

## References

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