Upper powerdomains of quasicontinuous dcpos

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Powerdomain in domain theory plays an important role in modeling the semantics of nondeterministic functional programming languages (see [1, 3, 6]). Upper powerdomain (aka Smyth powerdomain) is one of the most classic powerdomains in domain theory. For a continuous domain, its upper powerdomain is order isomorphic to the set of all nonempty Scott compact upper subsets with the reversed inclusion order (e.g., see [1, 3]). However, the construction of upper powerdomain over a general directed complete poset (*dcpo* for short) is not clear for a long time (see [1, 7]).

In this talk, we give a topological construction of the upper powerdomain over any dcpo by using tools from *monotone determined spaces* [2] (aka *directed spaces* [9]) and Scott-completion [10]. We provide several equivalent conditions for the continuity of the upper powerdomain of quasicontinuous dcpos. We also give an example to show that, even for a quasicontinuous dcpo, its upper powerdomain is not continuous, and hence is not order isomorphic to the set of all nonempty Scott compact upper subsets with the reversed inclusion order. Furthermore, we introduce a class of special order structures, called strongly quasicontinuous domains, and show that the upper power domain over any strongly quasicontinuous domain is isomorphic to the set of all nonempty Scott compact upper subsets ordered by the set-theoretic reverse inclusion, which is a continuous semilattice.

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