Betweenness and equidistance in metric spaces

AISLING MCCLUSKEY*

University of Galway, Ireland aisling.mccluskey@universityofgalway.ie

The classic notions of betweenness and equidistance in Euclidean geometry readily generalise to the context of metric spaces. In this talk, we consider these notions from an axiomatic perspective, then analyze the role various axioms play when interpreted in a metric space – especially one where the metric is induced by a vector space norm.

The points lying between two given points x and y constitute the betweenness interval I(x, y) bracketed by x and y, and the points equidistant from two given points x, y constitute the equiset E(x, y) with x, y as cocenters. An equiset naturally gives rise to a division of the underlying space into two comparative nearness regions; in the case of the Euclidean plane, each such region is a half-plane bounded by the line that is the equiset.

Betweenness intervals naturally engender a notion of convexity, and this talk focusses on the issue of when equisets and their comparative nearness regions, as well as the betweenness intervals themselves, are convex. In this regard, we provide some characterisations of normed vector space properties purely in terms of abstract betweenness, equidistance and comparative nearness.

References

 P. Bankston, A. McCluskey, Betweenness and equidistance in metric spaces. Journal of Convex Analysis, 30 (1), 371 - 400, 2023.

^{*}This is joint work with P. Bankston (Marquette University, Milwaukee, USA).