Automatic continuity and the Effros property

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All spaces (and groups) are assumed to be separable and metrizable. We will say that a group G is *Effros* if every continuous transitive action of G on a non-meager space X is micro-transitive (that is, the mapping $g \longmapsto gx$ is open for every $x \in X$). In [An], building on the classic paper [Ef], Ancel showed that every Polish group is Effros. In [vM], van Mill extended this result to all analytic groups. We will discuss the following results, which complete the picture:

- Under the Axiom of Determinacy, every group is Effros,
- Under the Axiom of Choice, there exists a non-Effros group,
- Under $\mathsf{V} = \mathsf{L}$, there exists a coanalytic non-Effros group.

The last two results are based on the (somewhat folklore) observation that the Effros property is sufficient to obtain several classical results concerning the automatic continuity of group homomorphisms. In fact, both of the above non-Effros groups will be graphs of discontinuous homomorphisms.

References

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