Products of uniform realcompactifications

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By a uniform realcompactification we mean a realcompactification of a uniform space (X, μ) defined by a certain family of real-valued uniformly continuous functions. In particular, a uniform realcompactification is always a topological subspace of the Samuel compactification $s_\mu X$, that is, the smallest compactification of the uniform space such that every bounded real-valued uniformly continuous function can be continuously extended to the compactification.

We will consider the following uniform realcompactifications of (X, μ):

1. $K(X)$, which is the G_δ-closure of X in its Samuel compactification $s_\mu X$;
2. the Samuel realcompactification $H(U_\mu(X))$, which is the smallest realcompactification of X such that every real-valued uniformly continuous function can be continuously extended to it;
3. $e_\mu X$, which is the completion of the uniform space $(X, e\mu)$ where $e\mu$ denotes the countable modification of the uniformity μ.

In general, all of these realcompactifications are different, and we are interested here in studying, for each one, the corresponding problem of the product. Namely, given two uniform spaces (X, μ) and (Y, ν), we want to find necessary and sufficient conditions in order to guarantee the following equivalences:

$$K(X \times Y) \approx K(X) \times K(Y)$$

$$H(U_{\mu\times\nu}(X \times Y)) \approx H(U_\mu(X)) \times H(U_\nu(Y))$$

$$e_{\mu\times\nu}(X \times Y) \approx e_\mu X \times e_\nu Y$$

While obtaining sufficient conditions is not very complicated, to find necessary conditions has not a very precise answer, in general.

Finally, we will also present a recent result obtained in [4], related to the corresponding problem of the product of the Lipschitz realcompactifications in the realm of metric spaces. Recall that, for a metric space (X, d), its Lipschitz realcompactification $H(Lip_d(X))$ is the smallest realcompactification of the metric space such that every real-valued Lipschitz function can be continuously extended to it.

References

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