Localic properties at infinity

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We say that a completely regular locale L has *property $\mathcal{P}$ at infinity* if $\beta L \setminus L$ has property $\mathcal{P}$, where βL is its Stone-Čech compactification. Regular continuous locales are precisely the ones that are compact at infinity. In this talk, we shall focus on the cases where $\mathcal{P}$ ranges through the following properties: connectedness, zero-dimensionality, σ-compactness, and Lindelöfness. In particular, we will show that a completely regular locale L is Lindelöf at infinity if and only if L is *rim-compact* (that is, L has a basis B such that the closed sublocale $\uparrow(b \vee b^*)$ is compact, for all $b \in B$).

References

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