Point-free Hausdorff axioms, and why the plural

Aleš Pultr *

Charles University, Prague, Czech Republic
pultr@kam.mff.cuni.cz

Separation axioms mimicking those of classical topology can be divided into three groups.

There are the "T_1-type" axioms among which the most suitable for point-free purposes is perhaps the *subfitness* – but also *fitness* (slightly mysterious, because it has some features of higher separation) and on the other hand (also, and perhaps more, mysterious) T_U, and plain T_1 are of interest, but it is another story.

On the other side there is the group of higher separation: regularity, complete regularity, normality (and stronger normalities), requirements that present no troubles in translating into the point-free context.

And there is the group of Hausdorff axioms in the middle, a colourful group of variously motivated natural conditions: the Dowker's and Strauss's formulas trying to be as similar to the pointy ones as possible and the Isbell's category motivated strong axiom, Johnstone and S. Shu-Hao motivated first with mending the non-conservativeness of the Isbell's approach and Paseka and Šmarda's quite differently motivated one (and the surprising confluence of the two), or Rosický and Šmarda's approach via natural strengthening the T_1.

We will tell some more about the history of such Hausdorff axioms, about their relations and natural requirements on their behaviour, and show that in concrete situations one has to choose between such requirements and conservativeness (agreement with the classical axiom when applied for spaces).

References

[1] J. Picado and A. Pultr, *Separation in point-free topology*, Birkhäuser/Springer Cham, 2021.

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