Equivariant extension operators

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In the category of topological spaces results about simultaneous extensions of continuous maps are well known. In [2] J. Dugundji established his famous extension theorem which is stated in the first two claims of the following simultaneous extension result.

Theorem 1 *Let A be a closed subset of a metrizable space Z and V a locally convex topological vector space. Let $C(Z,V)$ denote the locally convex vector space of all continuous maps $f : Z \to V$ endowed with the compact-open topology. Then there exists a linear operator $\varphi : C(A,V) \to C(Z,V)$ such that:*

1. *$\varphi(f)$ is an extension of f for every $f \in C(A,V)$,*
2. *$Im(\varphi(f)) \subset conv(Im(f))$,*
3. *φ is a linear homeomorphic embedding provided $C(A,V)$ and $C(Z,V)$ both carry the compact-open topology.*

Here $Im(f)$ stands for the image of the map f and *conv* denotes the convex hull. The last claim was proved by E. Michael [3, Theorem 7.1].

In this talk, we will explore a method for extending the above theorem to the category of G-spaces where G is a compact Lie group. We are going to present our new result about simultaneous extension operators recently published in [1].

Theorem 2 *Let G be a compact Lie group, Z a metrizable G-space, A a closed invariant subset of Z, and V a locally convex linear G-space. There exists an invariant neighborhood X of A in Z and an equivariant linear operator*

$$\Lambda : C(A,V) \to C(Z,V)$$

such that:

1. *$\Lambda(f)$ is an extension of f for every $f \in C(A,V)$,*
2. *$Im(\Lambda(f)) \subset conv(Im(f) \cup \{0\})$,*
3. *$Im(\Lambda(f)|_X) \subset conv(Im(f))$,*
4. *Λ is a linear equivariant homeomorphic embedding provided $C(A,V)$ and $C(Z,V)$ both carry compact-open topology and the linear action defined by $gf(x) = g^{-1}f(gx)$.*

We will also explain why in item 3 one cannot take $X = Z$.

References

[1] S. A. Antonyan, Ricardo Ramírez Luna, *Equivariant simultaneous extension operators for continuous maps*, Topol. Appl. 329 (2023) 108375.

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[3] E. Michael, *Some extensions theorems for continuous fuctions*, Pacific J. Math. 3 (1953), 789-806.

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