Closure Operators for Semitopogenous Spaces

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Semitopogenous orders on a set X were introduced by Császár to provide a unified approach to topology, proximity, and uniformity. Given a topology τ on X, one of the motivating examples is the semitopogenous order defined by $A \sqsubset U$ if and only if $A \subseteq \operatorname{int} U$. Thus, $A \sqsubset U$ may be used to model the idea that U is a neighborhood of A. Closure operators may now be defined from a semitopogenous order using the ideas that $x \in cl_{\sqsubset}(A)$ if and only if every neighborhood of x intersects A, or $x \in cl_{\sqsubset}(A)$ if and only if every neighborhood of x is a neighborhood of some point $a \in A$. A topology $\mathcal{T}_{\sqsubset}$ may be defined from $\sqsubset$ using the idea that U is open if it is a neighborhood of each of its points, and this topology gives a Kuratowski closure operator $cl_{\mathcal{T}_{\sqsubset}}$. A fourth closure operator $cl^{\sqsubset}$ may be defined for a semitopogeous order $\sqsubset$ as an analog of the kernel of A, that is, the intersection of all open sets containing A. We provide a systematic comparison of these closure operators. Examples are presented to show their relative dependence and independence.

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