Complete congruences of completely distributive lattices

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All the binomial lattices [1] embed into the quantale $Q(I)$ of sup-preserving endomaps of the unit interval. Elements of these lattices can be seen as monotone paths from $(0,0)$ to $(1,1)$, discrete paths for the binomial lattices, continuous paths for $Q(I)$ [2]. We aim at extending a natural geometric interpretation of lattice congruences of binomial lattices to congruences of $Q(I)$. This is, in particular, a completely distributive lattice.

Relying on Lawson-Hoffmann duality [3, 4], we characterise those maps between continuous domains that give rise to complete maps between completely distributive lattices. This allows to describe the complete congruences of an arbitrary completely distributive lattice by means of an interior operator on the collection of the closed sets of an associated topological space. In particular, we show that these congruences form a frame. We study this frame for the unit interval lattice, arguing that this frame is not a Boolean algebra, nor it is a (co)spatial. For the quantale $Q(I)$, we give a geometrical interpretation of these congruences by means of directed homotopies.

References

[1] Bennett, M.K., Birkhoff, G.: Two families of Newman lattices. Algebra Universalis **32**(1), 115–144 (1994).

[2] Santocanale, L., Gouveia, M.J.: The continuous weak order. Journal of Pure and Applied Algebra **225**, 106472 (2021).

[3] Lawson, J.D.: The duality of continuous posets. Houston J. Math. **5**, 357–386 (1979).

[4] Hoffmann, R.E.: Continuous posets, prime spectra of completely distributive complete lattices, and Hausdorff compactifications. Continuous lattices, Proc. Conf., Bremen 1979, Lect. Notes Math. 871, 159-208 (1981).

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