The T_D axiom in a pointfree setting

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A topological space X is T_D if, for each $x \in X$, there is an open set U containing x for which $U \setminus \{x\}$ is also open. In [2], Banaschewski and Pultr link this notion with pointfree topology by providing a dual equivalence between the category of T_D spaces and continuous maps, and the category of T_D-spatial frames and D-homomorphisms. Arrieta and Suarez, in [1], substantially extend this topic, using sublocales and the coframe of sublocales as fundamental tools.

We are interested in understanding the similarities and the differences between these ideas in the context of frames and spaces, and our context of partial frames and partial spaces (see below for some details). That things are not identical is immediately clear: in classical topology, the T_D axiom lies between T_0 and T_1, but there exists a partial space that is compact, regular, Hausdorff and yet not T_D.
We discuss appropriate tools, in particular, linked pairs, slicing points and D-homomorphisms, and use these to establish a dual equivalence between the category of T_D partial spaces and continuous maps, and the category of T_D-spatial partial frames and D-homomorphisms. We investigate the embedding of a partial frame into its free frame, and into its congruence frame, showing that the behaviour of slicing points differs from that of (ordinary) points.

Our broad context:

Partial frames are meet-semilattices where, in contrast with frames, not all subsets need have joins. A selection function, S, specifies, for all meet-semilattices, certain subsets under consideration, which we call the "designated" ones; an S-frame then must have joins of (at least) all such subsets and binary meet must distribute over these. A small collection of axioms suffices to delineate the selection functions we wish to consider; these axioms are sufficiently general to include as examples of partial frames bounded distributive lattices, sigma-frames, kappa-frames and frames.
Partial spaces relate to partial frames in exactly the way spaces do to frames: the open set and spectrum functors provide an adjunction, in which the fixed objects are the sober partial spaces and the spatial partial frames.

References

[1] Arrieta, I. and Suarez, A.L., The coframe of D-sublocales of a locale and the T_D-duality, Topology and its Applications, 291 (2021) 107614.

[2] Banaschewski, B. and Pultr, A., Pointfree aspects of the T_D axiom of classical topology, Quaestiones Mathematicae, 33 (2010) 369 - 385.

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