

Quantale-valued maps and partial maps

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Let \mathbf{Q} be a commutative and unital quantale. A \mathbf{Q} -map between sets X and Y is a left adjoint in the quantaloid of sets and \mathbf{Q} -relations, and a partial \mathbf{Q} -map from X to Y is a \mathbf{Q} -map from X to $Y \amalg \{\star\}$, the disjoint union of Y and the singleton set. We show that the following statements are equivalent:

- (i) \mathbf{Q} is lean.
- (ii) Every \mathbf{Q} -map is the graph of a map in **Set**.
- (iii) The canonical functor from the category of sets and partial \mathbf{Q} -maps to the category of sets and \mathbf{Q} -maps is monadic.

In particular, the equivalence of (i) and (ii) generalizes [1, Proposition III.1.2.1] to the non-integral setting.

References

- [1] D. Hofmann, G. J. Seal, and W. Tholen, editors. *Monoidal Topology: A Categorical Approach to Order, Metric, and Topology*. Cambridge University Press, Cambridge, 2014.

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