Generically hereditarily equivalent Peano continua

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We say that a continuum X is a hereditarily equivalent continuum if every nondegenerate subcontinuum of it is homeomorphic to X. This concept is one of the main motivations behind the construction of the pseudo-arc. If considered in the hyperspace of continua of X, denoted by $\mathrm{Cont}(X)$, it means that

$$\mathrm{Cont}(X) \setminus \mathrm{Fin}(X) = \{K \in \mathrm{Cont}(X) \mid K \simeq X\}.$$

This is an open and dense set, hence comeager, thus we can say that the generic subcontinua of X is homeomorphic to X. Therefore, it is natural to ask if there exist other spaces that satisfy this weaker property of having such collection of homeomorphic sets being comeager. We call these spaces generically hereditarily equivalent continua and show that the generalized Wazewski dendrites W_M for $M \subseteq \{3, 4, \ldots, \infty\}$ are such spaces. Moreover, in the hyperspace of maximal order arcs of W_M, the chains having every nondegenerate element homeomorphic to W_M make a comeager subset of the maximal order arcs, a property shared with hereditarily equivalent continua. Finally, we show that it is possible to find a comeager collection of chains with even more specific properties.

References

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