Game-theoretic results in regards to cardinal functions

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Some of the most famous results regarding cardinal functions consist in using them to provide an upper bound to the cardinality of some classes of spaces. Classic results in this area include Arhangel'skii's theorem, which states that for each Hausdorff space X, $|X| \leq 2^{\chi(X)L(X)}$; Hajnal and Juhász' inequality: for Hausdorff spaces, $|X| \leq 2^{\chi(X)c(X)}$; and Bell, Ginsburg and Woods' theorem: for a normal space X, $|X| \leq 2^{\chi(X)wL(X)}$, thus presenting a common generalization to the previous two for the class of normal spaces.

These results have spawned a number of questions, such as Arhangel'skii's problem whether there is an upper bound to Lindelöf spaces with points G_δ and, in recent years, many authors have used infinite topological games to give partial answers to these problems. One such example is Scheepers and Tall's theorem, stating that if a space has points G_δ and player II has a winning strategy in the Rothberger game of length ω_1, then its cardinality is at most $2^{\aleph_0}$.

In this talk, we want to discuss some of our successes in refining previouisly known results and inequalities in this field by using the weak Rothberger game and the cellularity game, as well as provide a few examples which illustrate their sharpness whenever possible.

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