On the entropy paradox on tame graphs

Jakub Tomaszewski*

al. Mickiewicza 30 30-059 Kraków
tomaszew@agh.edu.pl

By the works of Harańczyk, Kwietniak and Oprocha it is well known that exact maps on unit interval as well as on topological graphs can attain lower entropy than pure mixing maps. This "entropy paradox" results with a dichotomy between quantifying chaos using entropy and the hierarchy based on dynamical complexity: transitivity, mixing and exactness.

In the case of topological trees the paradox fades away, meaning that pure mixing maps and exact maps share the same infima for attainable entropy. The same happens for dendrites, a natural extension of the previous case.

Now, the most interesting is the case of tame graphs (topological spaces such that the closure of the set of endpoints and branching points is countable), which are a family in-between topological graphs, topological trees and dendrites. We will examine the entropy paradox on tame graphs, showing that it also fades away. We will consider both tame graphs with loops and without them, proving that it is not a decisive factor.

References

[1] Grzegorz Harańczyk, Dominik Kwietniak, *When lower entropy imples stronger Devaney chaos*, Proceedings of the American Mathematical Society, Volume 137, Number 6, June 2009, pp. 2063–2073.

[2] Grzegorz Harańczyk, Dominik Kwietniak, Piotr Oprocha, *Topological structure and entropy of mixing graph maps*, Ergodic Theory and Dynamical Systems, Volume 34, Issue 05, October 2014, pp. 1587-1614.

[3] Dominik Kwietniak, Piotr Oprocha, Jakub Tomaszewski, *Entropy of pure mixing maps on dendrites*, preprint, 2024.

*Joint work with Dominik Kwietniak (Jagiellonian University) and Piotr Oprocha (AGH University).