A topological bottleneck in Quantum Information

Stuart Wayland*

1156 High Street Santa Cruz, CA 95064
swayland@ucsc.edu

A central element in quantum information processing is the Bloch Sphere - an application of the topological equivalence between $\mathbb{C}P^1$ (the complex projective line) and S^2 (the 2 sphere $S^2 \subseteq \mathbb{R}^3$). This particularly convenient representation of the space of 2-dimensional *quantum states* has allowed for significant progress in algorithmic design for quantum optimization problems [1]. Recently there has been new motivation to expand this understanding to higher dimensional quantum states. However, the space of d-dimensional quantum states, isomorphic to the $2(d-1)$ dimensional manifold $\mathbb{C}P^{d-1}$, has a far richer topology than a sphere in $\mathbb{R}^{d^2-1}$ (in the case that $d > 2$) [2]. Constructing a new representation of $\mathbb{C}P^{d-1}$ from a basis of *quantum states* (trace 1, PSD, Hermitian matrices) that preserves its topological symmetries will significantly contribute to improving optimization of quantum problems.

References

[1] C. Carlson, Z. Jorquera, A. Kolla, S. Kordonowy, and S. Wayland, "Approximation Algorithms for Quantum Max-d-Cut." Sep. 19, 2023.

[2] I. Bengtsson and K. Zyczkowski, "GEOMETRY OF QUANTUM STATES: An Introduction to Quantum Entanglement".

*This is joint work with Professor Alexandra Kolla, Juspreet Singh Sandhu, Zackary Jorquera, and Steven Kordonowy (University of California, Santa Cruz).