Orbit structure in CR-dynamical systems

Andrew Wood*

University of Auckland
awoo337@aucklanduni.ac.nz

A CR-dynamical system is a pair (X, G), where X is a non-empty compact metric space, and G is a closed relation on X. In a classical dynamical system (X, f), the *trajectory* of a point x is the sequence $(x, f(x), f^2(x), \ldots)$. If the set of points in a trajectory, the *orbit*, is dense in X, then the point is a *transitive point*. CR-dynamical systems generalise classical dynamical systems, allowing for points to have multiple (or even no) trajectories. The resulting orbit structures can be complicated. Iztok Banič, Goran Erceg, Sina Greenwood, and Judy Kennedy introduced three types of transitive points in CR-dynamical systems. We develop the notion of a transitivity tree to capture the orbit structure of a point in a CR-dynamical system. Transitivity trees provide a useful tool for studying trajectories and the transitivity properties. We employ them to show the relationship between the three types of transitive points, and we show, for example, that when there are isolated points and there are 2-transitive points, every 3-transitive point is 2-transitive. Furthermore, we introduce a fourth type of a transitive point, allowing us to generalise further results from transitive points in classical dynamics to CR-dynamical systems.

*This is joint work with Sina Greenwood (University of Auckland).