Filters on ω and convergence of measures on Boolean algebras

TOMASZ ŻUCHOWSKI

Mathematical Institute, University of Wrocław, Poland
tomasz.zuchowski@math.uni.wroc.pl

An infinite Boolean algebra $\mathcal{A}$ is said to have the Nikodym property when every sequence of measures $\langle \mu_n : n \in \omega \rangle$ on $\mathcal{A}$ such that $\mu_n(A) \to 0$ for all $A \in \mathcal{A}$ is uniformly bounded. For a free filter F on ω we consider the space $N_F = \omega \cup \{p_F\}$, where ω is a discrete subspace and open neighborhoods of p_F are of the form $X \cup \{p_F\}$ for $X \in F$.

We define a property of the filter F which implies that any Boolean algebra $\mathcal{A}$ cannot have the Nikodym property when N_F is homeomorphically embedded into the Stone space $St(\mathcal{A})$ of ultrafilters on $\mathcal{A}$. We characterize this property in terms of sequences of non-negative measures on ω, and in terms of exhaustive ideals associated to density submeasures on ω. Moreover, we study the structure of the Katětov preorder on this class of filters.

Our results also apply to the Grothendieck property of Boolean algebras, which is closely related to the Nikodym property. Using results of Marciszewski and Sobota [1] concerning the Grothendieck property and N_F spaces, we obtain large families of algebras with the Nikodym property but without the Grothendieck property

References

[1] W. Marciszewski, D. Sobota, *The Josefson–Nissenzweig theorem and filters on ω*, to appear in Arch. Math. Logic.

[2] T. Żuchowski, *The Nikodym property and filters on ω*, preprint (2024), arXiv:2403.07484.