r-skeletons and ω-monotone functions

Cenobio Yescas Aparicio

Universidad Tecnológica de la Mixteca, México

38th SUMMER CONFERENCE ON TOPOLOGY AND ITS APPLICATIONS
Coimbra, Portugal, July 2024

r-skeletons

A brief history...

A brief history...

Let T be a non-empty set,

A brief history...

Let T be a non-empty set,

- For $x \in \mathbb{R}^T$, let $supp(x) := \{t \in T : 0 \neq x_t\}$.

A brief history...

Let T be a non-empty set,

- For $x \in \mathbb{R}^T$, let $supp(x) := \{t \in T : 0 \neq x_t\}$.
- The Σ-***product*** of $\mathbb{R}^T$ is the subspace

A brief history...

Let T be a non-empty set,

- For $x \in \mathbb{R}^T$, let $supp(x) := \{t \in T : 0 \neq x_t\}$.
- The Σ-***product*** of $\mathbb{R}^T$ is the subspace

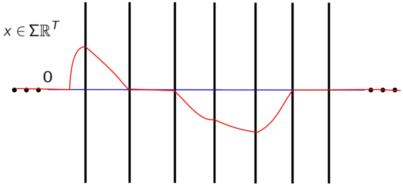
$$\Sigma_T := \left\{x \in \mathbb{R}^T : |supp(x)| \leq \aleph_0\right\}$$

A brief history...

Let T be a non-empty set,

- For $x \in \mathbb{R}^T$, let $supp(x) := \{t \in T : 0 \neq x_t\}$.
- The Σ-***product*** of $\mathbb{R}^T$ is the subspace

$$\Sigma_T := \left\{x \in \mathbb{R}^T : |supp(x)| \leq \aleph_0\right\}$$



Corson and Valdivia compact spaces

Corson and Valdivia compact spaces

X is a ***Corson*** compact space if $X \subseteq \Sigma_T$, for some T.

Corson and Valdivia compact spaces

X is a ***Corson*** compact space if $X \subseteq \Sigma_T$, for some T.

- The metrizable compact spaces are Corson compact spaces.

Corson and Valdivia compact spaces

X is a ***Corson*** compact space if $X \subseteq \Sigma_T$, for some T.

- The metrizable compact spaces are Corson compact spaces.

X is a ***Valdivia*** compact space if $Y = X \cap \Sigma_T$ is dense in X, for some T.

Corson and Valdivia compact spaces

X is a ***Corson*** compact space if $X \subseteq \Sigma_T$, for some T.

- The metrizable compact spaces are Corson compact spaces.

X is a ***Valdivia*** compact space if $Y = X \cap \Sigma_T$ is dense in X, for some T.

- The Corson compact spaces are Valdivia Compact spaces.

Corson and Valdivia compact spaces

X is a ***Corson*** compact space if $X \subseteq \Sigma_T$, for some T.

- The metrizable compact spaces are Corson compact spaces.

X is a ***Valdivia*** compact space if $Y = X \cap \Sigma_T$ is dense in X, for some T.

- The Corson compact spaces are Valdivia Compact spaces.
- The ordinal space $[0, \omega_1]$ is a Valdivia compact space.

r-skeletons

r-skeletons

Definition (Kubiś and Michalewski, 2006)

Let X be a compact space. An *r-skeleton* on X is a family $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ of retractions on X such that

r-skeletons

Definition (Kubiś and Michalewski, 2006)

Let X be a compact space. An *r-skeleton* on X is a family $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ of retractions on X such that

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

r-skeletons

Definition (Kubiś and Michalewski, 2006)

Let X be a compact space. An *r-skeleton* on X is a family $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ of retractions on X such that

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

r-skeletons

Definition (Kubiś and Michalewski, 2006)

Let X be a compact space. An *r-skeleton* on X is a family $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ of retractions on X such that

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow \subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $r_s(x) = \lim_{n\to\infty} r_{s_n}(x)$ for each $x \in X$; and

r-skeletons

Definition (Kubiś and Michalewski, 2006)

Let X be a compact space. An *r-skeleton* on X is a family $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ of retractions on X such that

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow\subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $r_s(x) = \lim_{n\to\infty} r_{s_n}(x)$ for each $x \in X$; and

(iv) for each $x \in X$, $x = \lim_{s\in\Gamma} r_s(x)$.

r-skeletons

r-skeletons

r-skeletons

- The *induced space* by $\{r_s\}_{s\in\Gamma}$ is the subset $\bigcup_{s\in\Gamma} r_s(X)$ of X.

r-skeletons

- The *induced space* by $\{r_s\}_{s\in\Gamma}$ is the subset $\bigcup_{s\in\Gamma} r_s(X)$ of X.

- If $\{r_s\}_{s\in\Gamma}$ is an r-skeleton on X, then we say that this r-skeleton

r-skeletons

- The *induced space* by $\{r_s\}_{s\in\Gamma}$ is the subset $\bigcup_{s\in\Gamma} r_s(X)$ of X.

- If $\{r_s\}_{s\in\Gamma}$ is an r-skeleton on X, then we say that this r-skeleton
  - it is *commutative* when $r_s \circ r_t = r_t \circ r_s$, for any $s, t \in \Gamma$; and

r-skeletons

- The *induced space* by $\{r_s\}_{s\in\Gamma}$ is the subset $\bigcup_{s\in\Gamma} r_s(X)$ of X.

- If $\{r_s\}_{s\in\Gamma}$ is an r-skeleton on X, then we say that this r-skeleton
  - it is *commutative* when $r_s \circ r_t = r_t \circ r_s$, for any $s, t \in \Gamma$; and
  - it is a *full* r-skeleton if $X = \bigcup_{s\in\Gamma} r_s(X)$.

Applications of r-skeletons.

Theorem (Kubiś and Michalewski,2006)

A compact space X is Valdivia iff it admits a commutative r-skeleton.

Applications of r-skeletons.

Theorem (Kubiś and Michalewski,2006)

A compact space X is Valdivia iff it admits a commutative r-skeleton.

Deduced from results of Bandlow and Kubiś:

Theorem (Cuth, 2014)

A compact space X is Corson compact space iff it admits a full r-skeleton.

Applications of r-skeletons.

Theorem (Kubiś and Michalewski,2006)

A compact space X is Valdivia iff it admits a commutative r-skeleton.

Deduced from results of Bandlow and Kubiś:

Theorem (Cuth, 2014)

A compact space X is Corson compact space iff it admits a full r-skeleton.

$$
\begin{array}{ccccc}
\text{Corson} & \longrightarrow & \text{Valdivia} & \longrightarrow & r\text{-skeleton} \\
 & \not\longleftarrow & & \not\longleftarrow & \\
 & [0,\omega_1] & & [0,\omega_2] &
\end{array}
$$

Applications of r-skeletons.

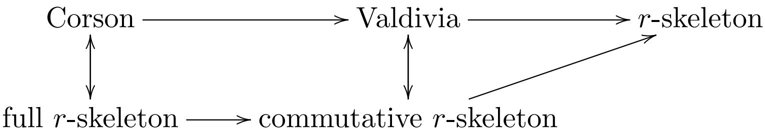
Theorem (Kubiś and Michalewski,2006)

A compact space X is Valdivia iff it admits a commutative r-skeleton.

Deduced from results of Bandlow and Kubiś:

Theorem (Cuth, 2014)

A compact space X is Corson compact space iff it admits a full r-skeleton.



Applications of r-skeletons

Theorem (Hernández-Hernández and Rojas-Hernández, 2022)

If X is a countably compact space which admits a full r-skeleton, then X is proximal.

Applications of r-skeletons

Theorem (Hernández-Hernández and Rojas-Hernández, 2022)

If X is a countably compact space which admits a full r-skeleton, then X is proximal.

Question (Islas-Jardon, 2015)

If X is a scattered Eberlein space, is then $\mathcal{K}(X)$ a semi-Eberlein space?

Applications of r-skeletons

Theorem (Hernández-Hernández and Rojas-Hernández, 2022)

If X is a countably compact space which admits a full r-skeleton, then X is proximal.

Question (Islas-Jardon, 2015)

If X is a scattered Eberlein space, is then $\mathcal{K}(X)$ a semi-Eberlein space?

Theorem (Correa,Cuth and Somaglia; 2022)

A compact space X is semi-Eberlein iff there exist an r-skeleton $\{r_s\}_{s\in\Gamma}$ on X with induced space Y, a bounded set $A \subset C(X)$ separating points of X and $D \subset Y$ dense in X such that:

(a) *$\{r_s\}_{s\in\Gamma}$ is A-shrinking with respect to D and*

(b) *$\lim_{s\in\Gamma'} r_s(x) \in D$ for all $x \in D$ and every up-directed subset $\Gamma' \subset \Gamma$.*

Applications of r-skeletons

Theorem (Hernández-Hernández and Rojas-Hernández, 2022)

If X is a countably compact space which admits a full r-skeleton, then X is proximal.

Question (Islas-Jardon, 2015)

If X is a scattered Eberlein space, is then $\mathcal{K}(X)$ a semi-Eberlein space?

Theorem (Rojas-Hernandez, Tenorio and Y-A, 2023)

If X is a semi-Eberlein space, then $\mathcal{K}(X)$ is a semi-Eberlein space.

ω-monotone functions

ω-monotone functions

ω-monotone functions

Definition (V.V. Tkachuk, 2014)

For up-directed σ-complete partially ordered sets Γ and Γ', a function $\varphi : \Gamma \to \Gamma'$ is called *ω-monotone* provided that:

ω-monotone functions

Definition (V.V. Tkachuk, 2014)

For up-directed σ-complete partially ordered sets Γ and Γ', a function $\varphi : \Gamma \to \Gamma'$ is called *ω-monotone* provided that:

1. if $s, t \in \Gamma$ and $s \leq t$, then $\varphi(s) \leq \varphi(t)$; and

ω-monotone functions

Definition (V.V. Tkachuk, 2014)

For up-directed σ-complete partially ordered sets Γ and Γ', a function $\varphi : \Gamma \to \Gamma'$ is called *ω-monotone* provided that:

1. if $s, t \in \Gamma$ and $s \leq t$, then $\varphi(s) \leq \varphi(t)$; and
2. if $\{s_n\}_{n<\omega} \uparrow \subseteq \Gamma$, then $\varphi(\sup_{n<\omega} s_n) = \sup_{n<\omega} \varphi(s_n)$.

Properties..

Properties..

If $f : X \to Y$ is a function, then $\varphi_f : [X]^{\leq\omega} \to [Y]^{\leq\omega}$ defined by $\varphi_f(A) = f(A)$, for each $A \in [X]^{\leq\omega}$, is an ω-monotone function.

Properties..

If $f : X \to Y$ is a function, then $\varphi_f : [X]^{\leq\omega} \to [Y]^{\leq\omega}$ defined by $\varphi_f(A) = f(A)$, for each $A \in [X]^{\leq\omega}$, is an ω-monotone function.

If $\varphi_1 : \Gamma \to \Gamma'$ and $\varphi_2 : \Gamma' \to \Gamma''$ are ω-monotone functions, then $\varphi_2 \circ \varphi_1$ is an ω-monotone function.

Properties..

If $f : X \to Y$ is a function, then $\varphi_f : [X]^{\leq\omega} \to [Y]^{\leq\omega}$ defined by $\varphi_f(A) = f(A)$, for each $A \in [X]^{\leq\omega}$, is an ω-monotone function.

If $\varphi_1 : \Gamma \to \Gamma'$ and $\varphi_2 : \Gamma' \to \Gamma''$ are ω-monotone functions, then $\varphi_2 \circ \varphi_1$ is an ω-monotone function.

If for $x \in X$ there is an assingment $s_x \in \Gamma$, then there exists an ω-monotone function $\varphi : [X]^{\leq\omega} \to \Gamma$ such that $s_x \leq \varphi(\{x\})$, for each $x \in X$.

r-skeletons and ω-monotone functions

r-skeletons and ω-monotone functions

Theorem

Let X be a compact space. If X admits an r-skeleton $\{r_s : s \in \Gamma\}$ with induced space Y, then X admits an r-skeleton $\{r_A : A \in [Y]^{\leq\omega}\}$ with induced space Y and $A \subseteq r_A(X)$, for every $A \in [Y]^{\leq\omega}$.

r-skeletons and ω-monotone functions

Theorem

Let X be a compact space. If X admits an r-skeleton $\{r_s : s \in \Gamma\}$ with induced space Y, then X admits an r-skeleton $\{r_A : A \in [Y]^{\leq\omega}\}$ with induced space Y and $A \subseteq r_A(X)$, for every $A \in [Y]^{\leq\omega}$.

O. Kalenda, 2009

If X is a Valdivia compact space, $A \subset X$, is the Alexandroff duplicate of X, $D(X, A)$, Valdivia?

r-skeletons and ω-monotone functions

Theorem

Let X be a compact space. If X admits an r-skeleton $\{r_s : s \in \Gamma\}$ with induced space Y, then X admits an r-skeleton $\{r_A : A \in [Y]^{\leq\omega}\}$ with induced space Y and $A \subseteq r_A(X)$, for every $A \in [Y]^{\leq\omega}$.

O. Kalenda, 2009

If X is a Valdivia compact space, $A \subset X$, is the Alexandroff duplicate of X, $D(X, A)$, Valdivia?

Theorem (García-Ferreria and Y-A, 2021)

Given $A \subseteq X$, the space $D(X, A)$ admits an r-skeleton iff there is an r-skeleton on X with induced space Y and an ω-monotone function $\delta : [A \setminus Y] \to [Y]^{\leq\omega}$ such that for each $B \in [A \setminus Y]^{\leq\omega}$, B is discrete in $X \setminus Y$ and $\overline{B} \setminus B \subseteq \overline{\delta(B)}$.

r-skeletons and ω-monotone functions

Theorem

Let X be a compact space. If X admits an r-skeleton $\{r_s : s \in \Gamma\}$ with induced space Y, then X admits an r-skeleton $\{r_A : A \in [Y]^{\leq\omega}\}$ with induced space Y and $A \subseteq r_A(X)$, for every $A \in [Y]^{\leq\omega}$.

O. Kalenda, 2009

If X is a Valdivia compact space, $A \subset X$, is the Alexandroff duplicate of X, $D(X, A)$, Valdivia?

Theorem (García-Ferreria and Y-A, 2021)

Given $A \subseteq X$, the space $D(X, A)$ admits an r-skeleton iff there is an r-skeleton on X with induced space Y and an ω-monotone function $\delta : [A \setminus Y] \to [Y]^{\leq\omega}$ such that for each $B \in [A \setminus Y]^{\leq\omega}$, B is discrete in $X \setminus Y$ and $\overline{B} \setminus B \subseteq \overline{\delta(B)}$.

r-skeletons and ω-monotone functions

Consider an r-skeleton $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ on X.

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow\subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $r_s(x) = \lim_{n\to\infty} r_{s_n}(x)$ for each $x \in X$; and

(iv) for each $x \in X$, $x = \lim_{s\in\Gamma} r_s(x)$.

r-skeletons and ω-monotone functions

Consider an r-skeleton $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ on X. For $s \in \Gamma$, let $F_s = r_s(X)$.

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow\subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $r_s(x) = \lim_{n\to\infty} r_{s_n}(x)$ for each $x \in X$; and

(iv) for each $x \in X$, $x = \lim_{s\in\Gamma} r_s(x)$.

r-skeletons and ω-monotone functions

Consider an r-skeleton $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ on X. For $s \in \Gamma$, let $F_s = r_s(X)$.

(i) F_s is a metrizable space, for each $s \in \Gamma$

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow\subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $r_s(x) = \lim_{n\to\infty} r_{s_n}(x)$ for each $x \in X$; and

(iv) for each $x \in X$, $x = \lim_{s\in\Gamma} r_s(x)$.

r-skeletons and ω-monotone functions

Consider an r-skeleton $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ on X. For $s \in \Gamma$, let $F_s = r_s(X)$.

(i) F_s is a metrizable space, for each $s \in \Gamma$

(ii) $F_s \subseteq F_t$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow\subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $r_s(x) = \lim_{n\to\infty} r_{s_n}(x)$ for each $x \in X$; and

(iv) for each $x \in X$, $x = \lim_{s\in\Gamma} r_s(x)$.

r-skeletons and ω-monotone functions

Consider an r-skeleton $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ on X. For $s \in \Gamma$, let $F_s = r_s(X)$.

(i) F_s is a metrizable space, for each $s \in \Gamma$

(ii) $F_s \subseteq F_t$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow\subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $F_s = \overline{\bigcup_{n<\omega} F_{s_n}}$; and

(iv) for each $x \in X$, $x = \lim_{s\in\Gamma} r_s(x)$.

r-skeletons and ω-monotone functions

Consider an r-skeleton $\mathcal{R} = \{r_s\}_{s\in\Gamma}$ on X. For $s \in \Gamma$, let $F_s = r_s(X)$.

(i) F_s is a metrizable space, for each $s \in \Gamma$

(ii) $F_s \subseteq F_t$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow\subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $F_s = \overline{\bigcup_{n<\omega} F_{s_n}}$; and

(iv) $X = \overline{\bigcup_{s\in\Gamma} F_s}$.

r-skeletons and ω-monotone functions

We have a family $\{F_s : s \in \Gamma\}$.

(i) F_s is a metrizable space, for each $s \in \Gamma$;

(ii) $F_s \subseteq F_t$ whenever $s \leq t$

(iii) if $\{s_n\}_{n<\omega} \uparrow \subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $F_s = \overline{\bigcup_{n<\omega} F_{s_n}}$; and

(iv) $X = \overline{\bigcup_{s\in\Gamma} F_s}$.

r-skeletons and ω-monotone functions

We have a family $\{F_s : s \in \Gamma\}$. For $s \in \Gamma$, we need a retraction $r_s : X \to X$ such that $F_s = r_s(X)$.

(i) F_s is a metrizable space, for each $s \in \Gamma$;

(ii) $F_s \subseteq F_t$ whenever $s \leq t$

(iii) if $\{s_n\}_{n<\omega} \uparrow \subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $F_s = \overline{\bigcup_{n<\omega} F_{s_n}}$; and

(iv) $X = \overline{\bigcup_{s\in\Gamma} F_s}$.

r-skeletons and ω-monotone functions

We have a family $\{F_s : s \in \Gamma\}$. For $s \in \Gamma$, we need a retraction $r_s : X \to X$ such that $F_s = r_s(X)$.

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $F_s \subseteq F_t$ whenever $s \leq t$

(iii) if $\{s_n\}_{n<\omega} \uparrow \subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $F_s = \overline{\bigcup_{n<\omega} F_{s_n}}$; and

(iv) $X = \overline{\bigcup_{s\in\Gamma} F_s}$.

r-skeletons and ω-monotone functions

We have a family $\{F_s : s \in \Gamma\}$. For $s \in \Gamma$, we need a retraction $r_s : X \to X$ such that $F_s = r_s(X)$.

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow \subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $F_s = \overline{\bigcup_{n<\omega} F_{s_n}}$; and

(iv) $X = \overline{\bigcup_{s\in\Gamma} F_s}$.

r-skeletons and ω-monotone functions

We have a family $\{F_s : s \in \Gamma\}$. For $s \in \Gamma$, we need a retraction $r_s : X \to X$ such that $F_s = r_s(X)$.

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow \subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $r_s(x) = \lim_{n\to\infty} r_{s_n}(x)$ for each $x \in X$; and

(iv) $X = \overline{\bigcup_{s\in\Gamma} F_s}$.

r-skeletons and ω-monotone functions

We have a family $\{F_s : s \in \Gamma\}$. For $s \in \Gamma$, we need a retraction $r_s : X \to X$ such that $F_s = r_s(X)$.

(i) $r_s(X)$ is a metrizable space, for each $s \in \Gamma$;

(ii) $r_s = r_s \circ r_t = r_t \circ r_s$ whenever $s \leq t$;

(iii) if $\{s_n\}_{n<\omega} \uparrow \subseteq \Gamma$ and $s = \sup\{s_n\}_{n<\omega}$, then $r_s(x) = \lim_{n\to\infty} r_{s_n}(x)$ for each $x \in X$; and

(iv) for each $x \in X$, $x = \lim_{s\in\Gamma} r_s(x)$.

c-skeletons

c-skeletons

Definition (Casarrubias et al., 2017)

For a space (X, τ). The pair $(\{F_s : s \in \Gamma\}, \varphi : \Gamma \to [\tau]^{\leq\omega})$, where $\{F_s : s \in \Gamma\}$ is a family of closed subsets of X, is a *c-skeleton* if:

c-skeletons

Definition (Casarrubias et al., 2017)

For a space (X, τ). The pair $(\{F_s : s \in \Gamma\}, \varphi : \Gamma \to [\tau]^{\leq\omega})$, where $\{F_s : s \in \Gamma\}$ is a family of closed subsets of X, is a *c-skeleton* if:

1. $F_s \subseteq F_t$, whenever$s \leq t$;
2. if $s \in \Gamma$, $\varphi(s)$ is a base for a topology τ_s on X and there exists a continuous map $g_s : (X, \tau_s) \to Z_s$ such that Z_s is a Tychonoff space and $g_s \restriction_{F_s}$ is a injective function;
3. φ is ω-monotone, and
4. $\bigcup_{s\in\Gamma} F_s$ is dense in X.

c-skeletons

Definition (Casarrubias et al., 2017)

For a space (X, τ). The pair $(\{F_s : s \in \Gamma\}, \varphi : \Gamma \to [\tau]^{\leq\omega})$, where $\{F_s : s \in \Gamma\}$ is a family of closed subsets of X, is a *c-skeleton* if:

1. $F_s \subseteq F_t$, whenever$s \leq t$;
2. if $s \in \Gamma$, $\varphi(s)$ is a base for a topology τ_s on X and there exists a continuous map $g_s : (X, \tau_s) \to Z_s$ such that Z_s is a Tychonoff space and $g_s \restriction_{F_s}$ is a injective function;
3. φ is ω-monotone, and
4. $\bigcup_{s\in\Gamma} F_s$ is dense in X.

The c-skeleton is *full* if $X = \bigcup_{s\in\Gamma} F_s$.

c-skeletons

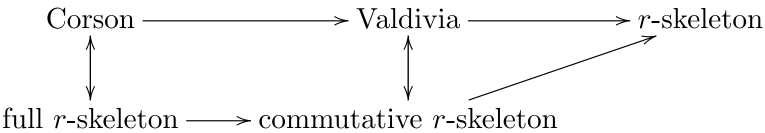
Theorem (Casarrubias et al., 2017)

A compact space X is Corson iff X admits a full c-skeleton.

c-skeletons

Theorem (Casarrubias et al., 2017)

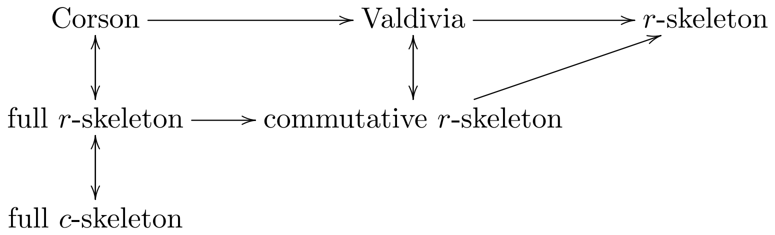
A compact space X is Corson iff X admits a full c-skeleton.



c-skeletons

Theorem (Casarrubias et al., 2017)

A compact space X is Corson iff X admits a full c-skeleton.



c-skeletons

Theorem (Casarrubias et al., 2017)

A compact space X is Corson iff X admits a full c-skeleton.

Theorem (Hernández-Hernández and Rojas-Hernández, 2022)

Let X be a compact space. X is a Corson space iff there exists an ω-monotone function $\sigma : [X]^{\leq\omega} \to [C_p(X)]^{\leq\omega}$ such that $\sigma(A)$ separates the points of $\mathrm{cl}_X(A)$ for each $A \in [X]^{\leq\omega}$.

c-skeletons

c-skeletons

Rojas-Hernández, Tenorio and Y-A, 2022

X admits a c-skeleton with induced space Y iff there exists an ω-monotone function $\sigma : [Y]^{\leq\omega} \to [C_p(X)]^{\leq\omega}$ such that $\sigma(A)$ separates the points of $\mathrm{cl}_X(A)$ for each $A \in [Y]^{\leq\omega}$.

c-skeletons

Rojas-Hernández, Tenorio and Y-A, 2022

X admits a c-skeleton with induced space Y iff there exists an ω-monotone function $\sigma : [Y]^{\leq\omega} \to [C_p(X)]^{\leq\omega}$ such that $\sigma(A)$ separates the points of $\mathrm{cl}_X(A)$ for each $A \in [Y]^{\leq\omega}$.

Rojas-Hernández, Tenorio and Y-A, 2023

Given a space X and a dense subspace Y of X, we say that X admits a *c-skeleton with induced subspace* Y if there exists an ω-monotone function $\sigma : [Y]^{\leq\omega} \to [C_p(X)]^{\leq\omega}$ such that $\sigma(A)$ separates the points of $\mathrm{cl}_X(A)$ for each $A \in [Y]^{\leq\omega}$.
When $Y = X$ we say that the c-skeleton is *full*.

c-skeletons and hyperspaces

c-skeletons and hyperspaces

Theorem (Rojas-Hernández, Tenorio and Y-A, 2023)

If Y is induced by a c-skeleton on a space X, then $\mathcal{K}_M(Y)$ is induced by a c-skeleton on $\mathcal{K}(X)$.

c-skeletons and hyperspaces

Theorem (Rojas-Hernández, Tenorio and Y-A, 2023)

If Y is induced by a c-skeleton on a space X, then $\mathcal{K}_M(Y)$ is induced by a c-skeleton on $\mathcal{K}(X)$.

Theorem (Rojas-Hernández, Tenorio and Y-A, 2023)

If X admits a c-skeleton with induced subspace Y, then any subspace Z of X, satisfying that $Z \cap Y$ is dense in Z, admits a c-skeleton which induces $Z \cap Y$.

c-skeletons and hyperspaces

Theorem (Rojas-Hernández, Tenorio and Y-A, 2023)

If Y is induced by a c-skeleton on a space X, then $\mathcal{K}_M(Y)$ is induced by a c-skeleton on $\mathcal{K}(X)$.

Theorem (Rojas-Hernández, Tenorio and Y-A, 2023)

If X admits a c-skeleton with induced subspace Y, then any subspace Z of X, satisfying that $Z \cap Y$ is dense in Z, admits a c-skeleton which induces $Z \cap Y$.

Corollary

If Y is induced by a c-skeleton on a space X, then $\mathcal{F}(Y)$ is induced by a c-skeleton on $\mathcal{F}(X)$ and $\mathcal{F}_n(Y)$ is induced by a c-skeleton on $\mathcal{F}_n(X)$ for each positive integer n.

c-skeletons

c-skeletons

Proposition (Rojas-Hernández, Tenorio and Y-A, 2023)

Let X be a compact space. If X has a dense subset consisting of G_δ-points and $\mathcal{K}(X)$ admits a c-skeleton then X admits a c-skeleton.

c-skeletons

Proposition (Rojas-Hernández, Tenorio and Y-A, 2023)

Let X be a compact space. If X has a dense subset consisting of G_δ-points and $\mathcal{K}(X)$ admits a c-skeleton then X admits a c-skeleton.

Proposition (Rojas-Hernández, Tenorio and Y-A, 2023)

Let Y be a space admitting a full c-skeleton. If $f : X \to Y$ is continuous and bijective, then X admits a full c-skeleton.

c-skeletons

c-skeletons

Questions

c-skeletons

Questions

- Is it true that if $\mathcal{F}_n(X)$ admits a c-skeleton, for some positive integer n, then X^n admits a c-skeleton?

c-skeletons

Questions

- Is it true that if $\mathcal{F}_n(X)$ admits a c-skeleton, for some positive integer n, then X^n admits a c-skeleton?
- Is it true that if X^n admits a c-skeleton, for some positive integer n, then $\mathcal{F}_n(X)$ admits a c-skeleton?

c-skeletons

Questions

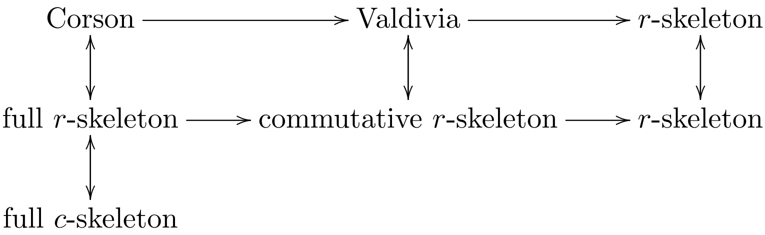
- Is it true that if $\mathcal{F}_n(X)$ admits a c-skeleton, for some positive integer n, then X^n admits a c-skeleton?
- Is it true that if X^n admits a c-skeleton, for some positive integer n, then $\mathcal{F}_n(X)$ admits a c-skeleton?
- Is there a space X such that X^2 admits a c-skeleton but such that X does not admit a c-skeleton?

Theorem (García-Ferreira and Y-A, 2023)

If X is a compact space, X admits an r-skeleton with induced space Y iff X admits a c-skeleton with induced space Y.

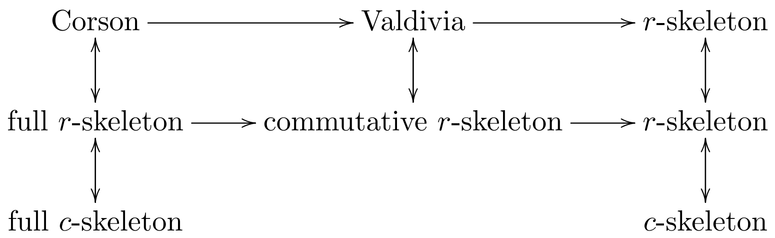
Theorem (García-Ferreira and Y-A, 2023)

If X is a compact space, X admits an r-skeleton with induced space Y iff X admits a c-skeleton with induced space Y.



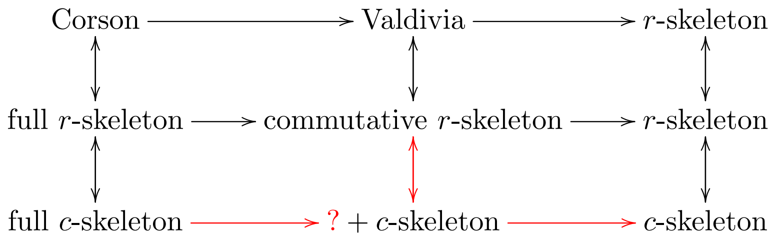
Theorem (García-Ferreira and Y-A, 2023)

If X is a compact space, X admits an r-skeleton with induced space Y iff X admits a c-skeleton with induced space Y.



Theorem (García-Ferreira and Y-A, 2023)

If X is a compact space, X admits an r-skeleton with induced space Y iff X admits a c-skeleton with induced space Y.



Relation with Valdivia compact spaces.

Question (Kalenda, 2009)

If $X \times Z$ is a Valdivia compact space, are X and Y Valdivia compact spaces?

Relation with Valdivia compact spaces.

Question (Kalenda, 2009)

If $X \times Z$ is a Valdivia compact space, are X and Y Valdivia compact spaces?
If X^2 is a Valdivia compact space, is X a Valdivia compact space?

Relation with Valdivia compact spaces.

Question (Kalenda, 2009)

If $X \times Z$ is a Valdivia compact space, are X and Y Valdivia compact spaces?
If X^2 is a Valdivia compact space, is X a Valdivia compact space?

If X^2 admits a commutative r-skeleton. Does X admit a commutative r-skeleton?

Relation with Valdivia compact spaces.

Question (Kalenda, 2009)

If $X \times Z$ is a Valdivia compact space, are X and Y Valdivia compact spaces?
If X^2 is a Valdivia compact space, is X a Valdivia compact space?

If X^2 admits a r-skeleton. Does X admit a r-skeleton?

Relation with Valdivia compact spaces.

Question (Kalenda, 2009)

If $X \times Z$ is a Valdivia compact space, are X and Y Valdivia compact spaces?
If X^2 is a Valdivia compact space, is X a Valdivia compact space?

If X^2 admits a c-skeleton. Does X admit a c-skeleton?

Relation with Valdivia compact spaces.

Question (Kalenda, 2009)

If $X \times Z$ is a Valdivia compact space, are X and Y Valdivia compact spaces?
If X^2 is a Valdivia compact space, is X a Valdivia compact space?

If X^2 admits a c-skeleton. Does X admit a c-skeleton?

Questions

- Is it true that if $\mathcal{F}_n(X)$ admits a c-skeleton, for some positive integer n, then X^n admits a c-skeleton?
- Is it true that if X^n admits a c-skeleton, for some positive integer n, then $\mathcal{F}_n(X)$ admits a c-skeleton?
- Is there a space X such that X^2 admits a c-skeleton but such that X does not admit a c-skeleton?

q-skeletons

Definition

A space X admits a *q-skeleton* if there exists a dense subspace Z of $C_p(X)$ and an ω-monotone function $\delta : [Z]^{\leq\omega} \to [X]^{\leq\omega}$ such that $\Delta_{\mathrm{cl}_{C_p(X)}(A)}(\delta(A))$ is a dense subspace of $\Delta_{\mathrm{cl}_{C_p(X)}(A)}(X)$ for each $A \in [Z]^{\leq\omega}$; in the case $Z = C_p(X)$ we say that the q-skeleton is *full*.

q-skeletons

Definition

A space X admits a *q-skeleton* if there exists a dense subspace Z of $C_p(X)$ and an ω-monotone function $\delta : [Z]^{\leq\omega} \to [X]^{\leq\omega}$ such that $\Delta_{\mathrm{cl}_{C_p(X)}(A)}(\delta(A))$ is a dense subspace of $\Delta_{\mathrm{cl}_{C_p(X)}(A)}(X)$ for each $A \in [Z]^{\leq\omega}$; in the case $Z = C_p(X)$ we say that the q-skeleton is *full*.

Theorem (Rojas-Hernández, Tenorio and Y-A, 2023)

If $C_p(X)$ admits a (full) c-skeleton, then X admits a (full) q-skeleton.

q-skeletons

Definition

A space X admits a *q-skeleton* if there exists a dense subspace Z of $C_p(X)$ and an ω-monotone function $\delta : [Z]^{\leq\omega} \to [X]^{\leq\omega}$ such that $\Delta_{\mathrm{cl}_{C_p(X)}(A)}(\delta(A))$ is a dense subspace of $\Delta_{\mathrm{cl}_{C_p(X)}(A)}(X)$ for each $A \in [Z]^{\leq\omega}$; in the case $Z = C_p(X)$ we say that the q-skeleton is *full*.

Theorem (Rojas-Hernández, Tenorio and Y-A, 2023)

A space $C_p(X)$ admits a (full) c-skeleton if and only if X admits a (full) q-skeleton.

Related problems

Related problems

- Find a description of Valdivia compact spaces via c-skeletons.

Related problems

- Find a description of Valdivia compact spaces via c-skeletons.
- Find a description of (semi)-Eberlein compact spaces via c-skeletons.

Thanks!! Obrigado!! Gracias!!