Akademia Górniczo-Hutnicza
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Graphs with tranches

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Warsaw circle

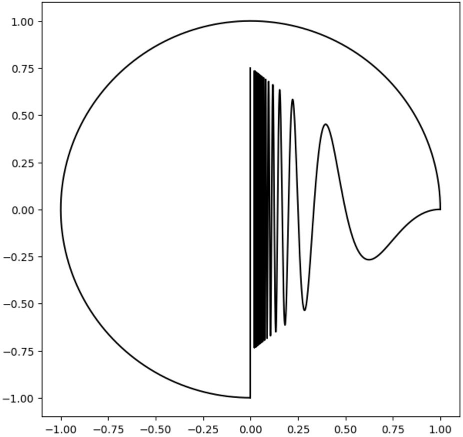


Figure: Warsaw Circle

Warsaw circle as a quasi-graph[1]

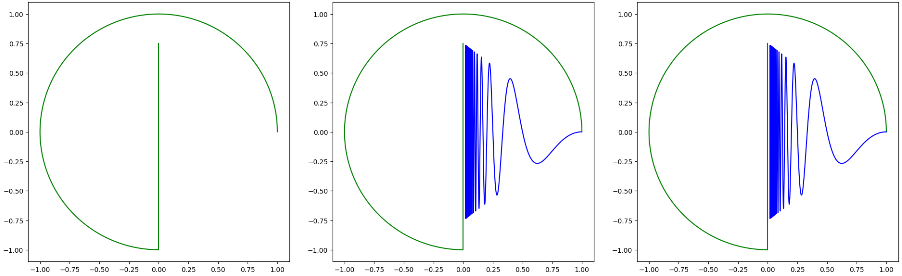
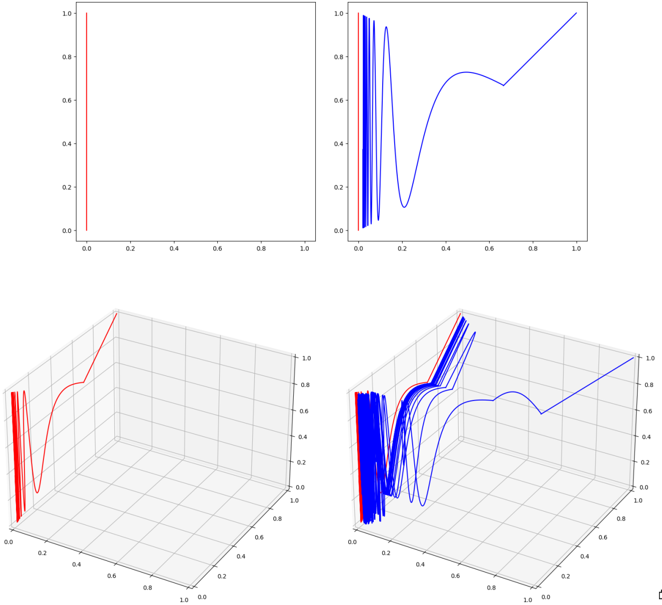


Figure: Construction of the Warsaw Circle as a quasi-graph

[1] E. Shi J. Mai. "Structures of quasi-graphs and ω-limit sets of quasi-graph maps". In: *Trans. Amer. Math. Soc.* 369.1 (2017), pp. 139–165. ISSN: 0002-9947. DOI: 10.1090/tran/6627. URL: https://doi.org/10.1090/tran/6627.

Higher order quasi-arcs



Warsaw circle as generalized sin(1/x)-type continuum[2]

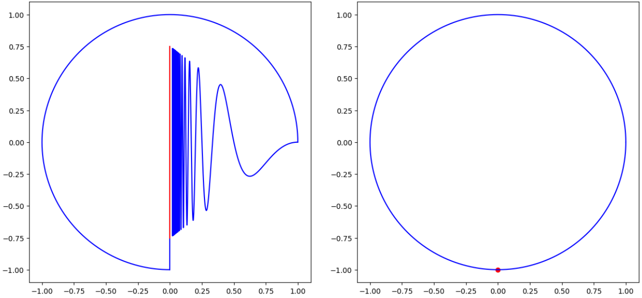


Figure: Warsaw Circle and its image under monotone mapping ϕ from definiton of sin(1/x)-type continuum

[2] C. Mouron L. Hoehn. "Hierarchies of chaotic maps on continua". In: *Ergodic Theory Dynam. Systems* 34.6 (2014), pp. 1897–1913. ISSN: 0143-3857. DOI: 10.1017/etds.2013.32. URL: https://doi.org/10.1017/etds.2013.32.

Quasi-graph thats not a sin(1/x)-type continuum

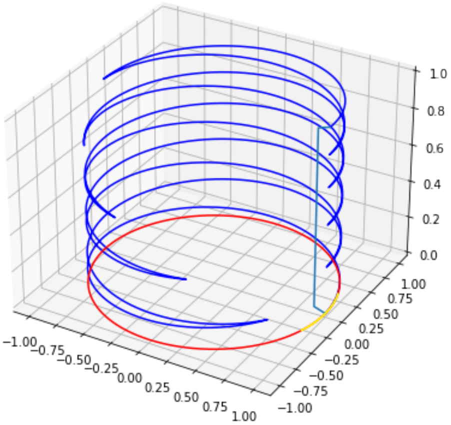


Figure: A quasi-graph whose limit set is circle, but is not a generalized sin(1/x)-type continuum

Sufficient conditon for being sin(1/x)-type continuum

Lemma

Let X be a quasi-graph. Then X is a regular tranched graph with mapping $\phi\colon X \to X/_{\sim}$, where relation $\sim$ collapses connected components of limit sets and ϕ is a natural projection.

Theorem

Let $X = G \cup \bigcup_{j=1}^{n} L_j$ be a quasi-graph. Assume that for every connected component $\Lambda \subset \bigcup \omega(L_j)$ the following assertions hold:

1. *There is a quasi-arc L in X such that $\omega(L) = \Lambda$ and*
2. *Continuum Λ is arc-like.*

Then X is a generalized sin(1/x)-type continuum.

Sin(1/x)-type continuum with branching point in a tranche

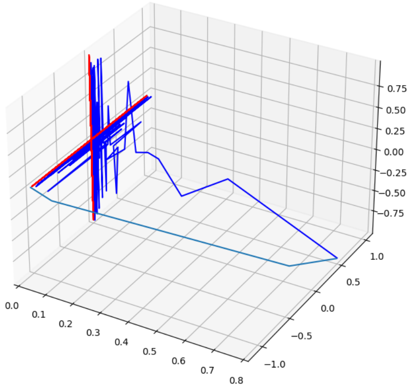


Figure: A quasi-graph which is generalized sin(1/x)-type continuum and contains 4-star as a tranch

Necessary conditon for being sin(1/x)-type continuum

Theorem

Let X be a quasi-graph that is a generalized sin(1/x) type continuum. Then for every connected component $\Lambda \subset \bigcup \omega(L_j)$ there is a quasi-arc $L \subset X$ such that $\omega(L) = \Lambda$

Set of tranches doesn't need to be closed

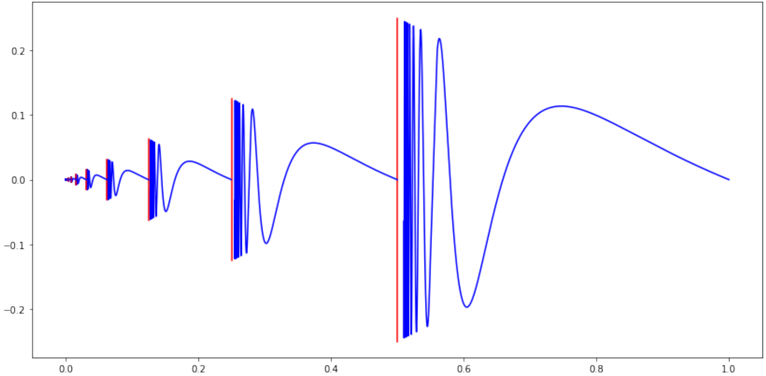
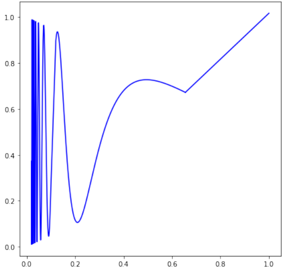


Figure: Generalized sin(1/x)-type continuum whose set of tranches is not closed

There can be infinite hierarchy of quasi-arcs

$$A = \bigcup_{n=0}^{\infty} \sigma^n(\{x, f(x), \ldots, f^n(x), \ldots) : x \in (0,1]\}) \cup \{0\}^{\infty}. \qquad (1)$$



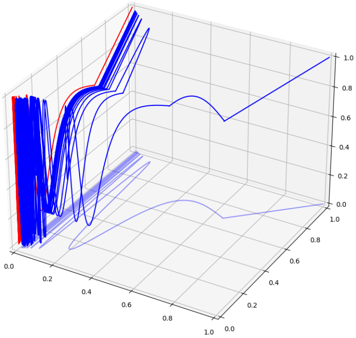


Figure: $f : (0,1] \to (0,1]$ and continuum of order 2

Double-sided sin(1/x)-continuum

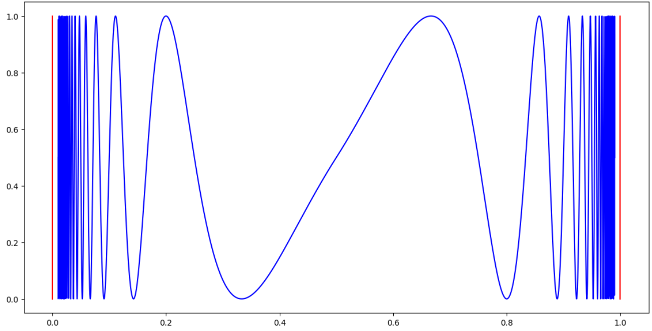


Figure: Continuum X

A strange sin(1/x)-type continuum

$$\widehat{X} = \{(x_0, x_1, \ldots, x_n, \ldots) : x_0 \in [0,1], \forall i \quad (x_{i-1}, x_i) \in X\}$$

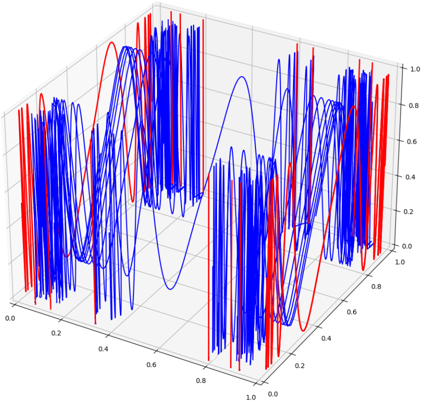


Figure: Projection of $\widehat{X}$ onto three dimensional space

Sufficient and necessary condition to be a quasi-graph.

We can prove that:

1. $\widehat{X}$ is a sin(1/x)-type continuum,
2. Set of tranches of $\widehat{X}$ is dense in $\widehat{X}$,
3. Every fiber is either a singleton or is homeomorphic to $\widehat{X}$,
4. Continuum $\widehat{X}$ contains no arcs inside it,

Theorem

Let X be a tranched graph. Then X is a quasi-graph if and only if it is arcwise connected, regular and of finite order.