Trivial self-homeomorphisms of ω^*

Will Brian

University of North Carolina at Charlotte

joint work with Ilijas Farah and Saeed Ghasemi

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$\beta\omega$ and ω^*

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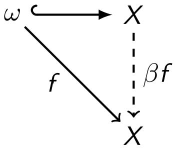
For example, the *shift map* $\sigma : \omega^* \to \omega^*$ is defined by setting

$$\sigma(u) = \text{the ultrafilter generated by } \{A+1 : A \in u\}.$$

In this way, σ is induced by the successor function $n \mapsto n+1$ on ω.

More generally, recall that $\beta\omega$ has the *Stone extension property*, which states that for any compact Hausdorff space X, any map $\omega \to X$ can be extended to a continuous map $\beta\omega \to X$.



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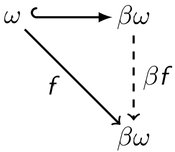

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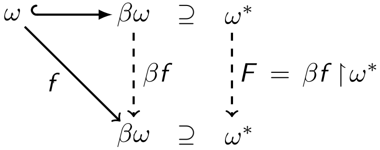

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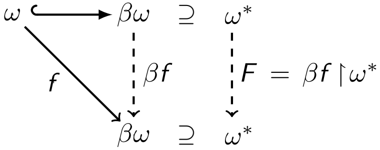

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Explicitly, for any ultrafilter $u \in \omega^*$, $F(u)$ is the ultrafilter generated by $\{f[A] : A \in u\}$.

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Building on Shelah's result, we now know that the forcing axiom OCA implies every self-homeomorphism of ω^* is trivial.

When are two maps the same?

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This kind of identity is studied in topological dynamics.

conjugation

Two dynamical systems (X, f) and (Y, g) are *conjugate* if there is a homeomorphism $h : X \to Y$ such that $h \circ f = g \circ h$.

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- Two self-homeomorphisms ϕ and ψ of ω^* are *conjugate* if there is an h as above with $h \circ \phi = \psi \circ h$.
- If, furthermore, there is a trivial self-homeomorphism h of ω^* with $h \circ \phi = \psi \circ h$, then ϕ and ψ are *trivially conjugate*.

Recall that it is consistent for all self-homeomorphisms of ω^* to be trivial. Thus "trivially conjugate" and "conjugate" mean the same thing in some models of set theory.

trivial maps can be non-trivially conjugate

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But it is also consistent that these notions do not coincide:

Theorem (Van Douwen, 1987)

The shift map σ and its inverse are not trivially conjugate. In particular, it is consistent that σ and σ^{-1} are not conjugate.

Theorem (B., 2024)

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Let us say two trivial maps are *potentially conjugate* if they can be made conjugate in a forcing extension (which is true if and only if CH implies they are conjugate).

more on what van Douwen said

The *prodigal index* of an almost bijection of ω is

$$\mathrm{Ind}(f) = |\mathrm{domain}(f) \setminus \mathrm{image}(f)| - |\mathrm{image}(f) \setminus \mathrm{domain}(f)|.$$

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Theorem (Van Douwen, 1987)

Suppose f and g are almost bijections of ω. If $\mathrm{Ind}(f) \neq \mathrm{Ind}(g)$, then the self-homeomorphisms of ω^* induced by f and g are not trivially conjugate.

Van Douwen's theorem reveals an obstruction to trivial conjugacy: If $\mathrm{Ind}(\phi) \neq \mathrm{Ind}(\psi)$, then ϕ and ψ are not trivially conjugate.

our main theorem

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Suppose ϕ and ψ are trivial self-homeomorphisms of ω^*. If $\mathrm{Ind}(\phi)$ and $\mathrm{Ind}(\psi)$ have different parities (one odd and one even), then ϕ and ψ are not conjugate.

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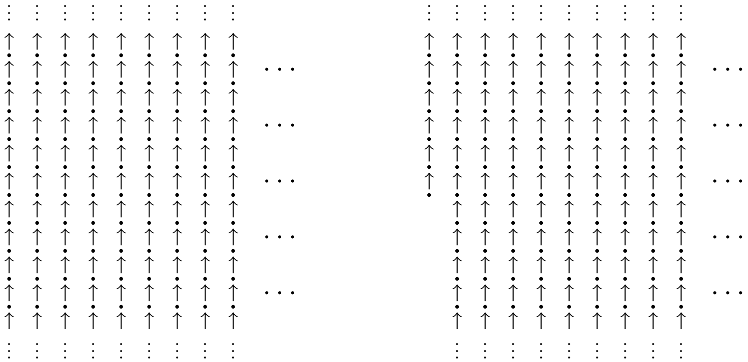
Theorem (BFG, 2024)

Suppose ϕ and ψ are trivial self-homeomorphisms of ω^*. Then ϕ and ψ are potentially conjugate if and only if they have the same prodigal index parity and the same first-order theory (i.e., the structures $\langle \omega^*, \phi \rangle$ and $\langle \omega^*, \psi \rangle$ are elementarily equivalent).

the main idea of the lemma

To illustrate the idea behind this lemma, consider the following two almost permutations of ω: one with infinitely many $\mathbb{Z}$-like orbits, and one with infinitely many $\mathbb{Z}$-like orbits plus one $\mathbb{N}$-like orbit.



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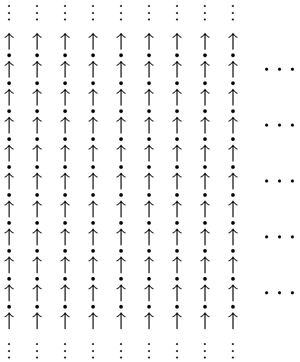

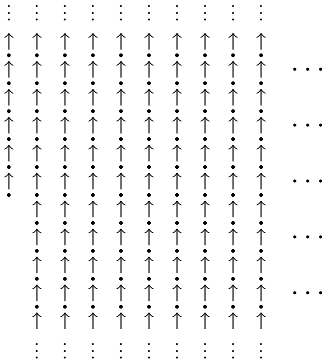

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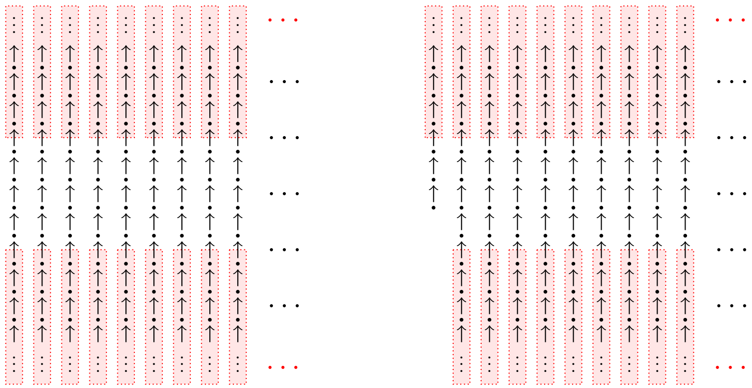
The clopen sets corresponding to the "ends" of these orbits are definable in the structure $\langle \omega^*, \phi \rangle$, where ϕ is the induced self-homeomorphism. They are identifiable as little copies of the shift (ω^*, σ).



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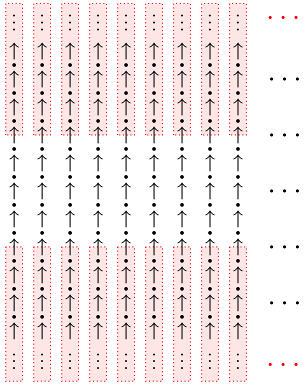

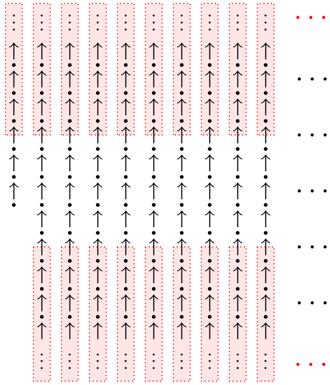

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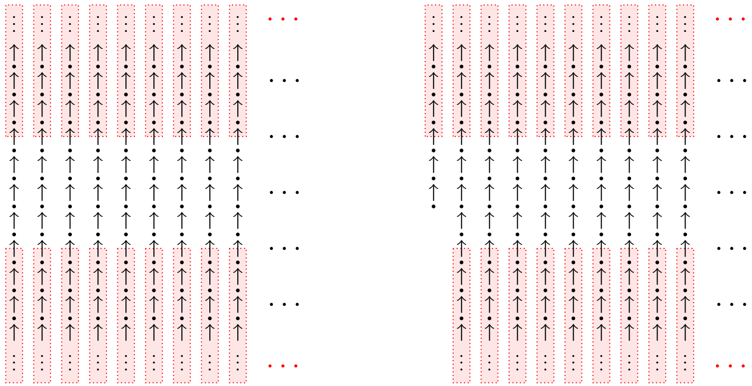

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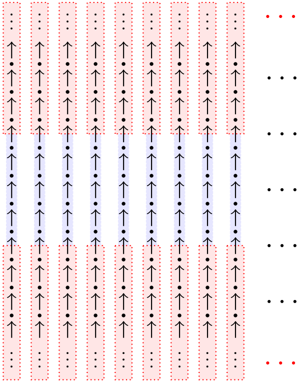

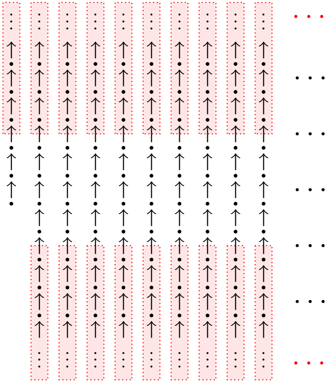

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Question

Suppose a non-trivial self-homeomorphisms of ω^* is elementarily equivalent to σ. Does CH *prove it is conjugate to σ?*

The end

Thank you for listening