Hofmann-Mislove through the lenses of Priestley

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It states that for each sober space X, the Scott-open filters of the frame $\mathcal{O}(X)$ of open subsets of X are (dually) isomorphic to the compact saturated subsets of X.

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Let $\boldsymbol{KSat}(X)$ be the poset of compact saturated subsets of X ordered by inclusion.

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Let $\boldsymbol{OFilt}(L)$ be the poset of Scott-open filters of L ordered by inclusion.

Hofmann-Mislove

Let X be a sober space and $L = O(X)$.
Then $OFilt(L)$ is isomorphic to $KSat(X)$.

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Priestley duality establishes a dual equivalence between the category of bounded distributive lattices and bounded lattice homomorphisms and the category of Priestley spaces and continuous order-preserving maps.

In particular, for each bounded distributive lattice L and Priestley dual Y of L, the Stone map φ is a bounded lattice isomorphism from L to the clopen upsets of Y.

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For a bounded distributive lattice D, let $\textbf{\textit{Filt}}(D)$ be the poset of filters of D ordered by inclusion.

For a Priestley space X, let $\textbf{\textit{ClUp}}(X)$ be the poset of closed upsets of X ordered by inclusion.

Let D be a bounded distributive lattice and Y its Priestley dual. Then $Filt(D)$ is isomorphic to $ClUp(Y)$.

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A close look at the two proofs reveals striking similarities.

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But they actually imply each other in full generality!

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The Stone map φ is an isomorphism of D to the lattice of compact open subsets of (Y, τ). Thus, φ is an isomorphism of D to a bounded sublattice of the frame L of open subsets of (X, τ).

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But $\mathbf{\mathit{KSat}}(X, \tau)$ is exactly $\mathbf{\mathit{ClUp}}(X)$, and $\mathbf{\mathit{OFilt}}(L)$ is isomorphic to $\mathbf{\mathit{Filt}}(D)$.

Thus, $\mathbf{\mathit{Filt}}(D)$ is dually isomorphic to $\mathbf{\mathit{ClUp}}(X)$, and Priestley's result follows.

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For simplicity, we identify X with its image and view X as a dense subset of Y.

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Theorem

F is Scott-open iff $\min K_F \subseteq X$.

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Corollaries

1. A Scott-open filter F is completely prime iff $\min K_F$ is a singleton.
2. Every Scott-open filter of L is an intersection of completely prime filters of L.

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Hofmann-Mislove is a direct consequence of the last two theorems.

Consequences

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For each frame L and the space X of points of L, we have that $\mathit{OFilt}(L)$ is dually isomorphic to $\mathit{KSat}(X)$.

This result is known, see e.g. (S. Vickers, 1989), but the proof relies on Zorn's lemma, while our proof needs only the Prime Ideal Theorem.

Dualities

$$
\begin{tikzcd}
\mathbf{Frm}^{op} \arrow[d, tail, no head] & \\
\mathbf{SFrm}^{op} \arrow[d, tail, no head] & \\
\mathbf{1SFrm}^{op} \arrow[dd, tail, no head] \arrow[dr, tail, no head] & \\
& \mathbf{ContFrm}^{op} \arrow[dd, tail, no head] \\
\mathbf{KSFrm}^{op} \arrow[dd, tail, no head] & \\
& \mathbf{StCFrm}^{op} \arrow[dl, tail, no head] \\
\mathbf{StKFrm}^{op} \arrow[dd, tail, no head] \arrow[dr, tail, no head] & \\
& \mathbf{CohFrm}^{op} \arrow[dd, tail, no head] \\
\mathbf{KRegFrm}^{op} \arrow[dr, tail, no head] & \\
& \mathbf{StoneFrm}^{op}
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathbf{Top}_0 \arrow[d, tail, no head] & \\
\mathbf{Sob} \arrow[d, tail, no head] & \\
\mathbf{GLCSob} \arrow[dd, tail, no head] \arrow[dr, tail, no head] & \\
& \mathbf{LCSob} \arrow[dd, tail, no head] \\
\mathbf{KSob} \arrow[dd, tail, no head] & \\
& \mathbf{SLCTop} \arrow[dl, tail, no head] \\
\mathbf{SCTop} \arrow[dd, tail, no head] \arrow[dr, tail, no head] & \\
& \mathbf{Spec} \arrow[dd, tail, no head] \\
\mathbf{KHaus} \arrow[dr, tail, no head] & \\
& \mathbf{Stone}
\end{tikzcd}
$$

A new proof of:

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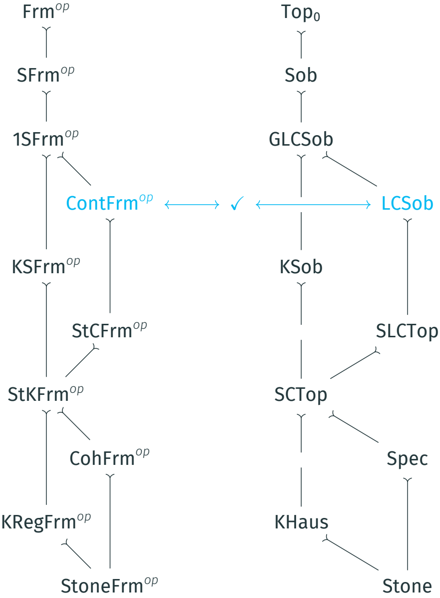

A new proof of:

- The Hofmann-Lawson duality between continuous frames and locally compact sober spaces

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\begin{tikzcd}
\mathsf{Frm}^{op} & & & \mathsf{Top}_0 & \\
\mathsf{SFrm}^{op} \arrow[u, tail] & & & \mathsf{Sob} \arrow[u, tail] & \\
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\mathsf{KSFrm}^{op} \arrow[uu, tail] & & & \mathsf{KSob} \arrow[uu, tail] & \\
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\mathsf{StKFrm}^{op} \arrow[uu, tail] \arrow[ur, tail] & & & \mathsf{SCTop} \arrow[uu, tail] \arrow[ur, tail] & \\
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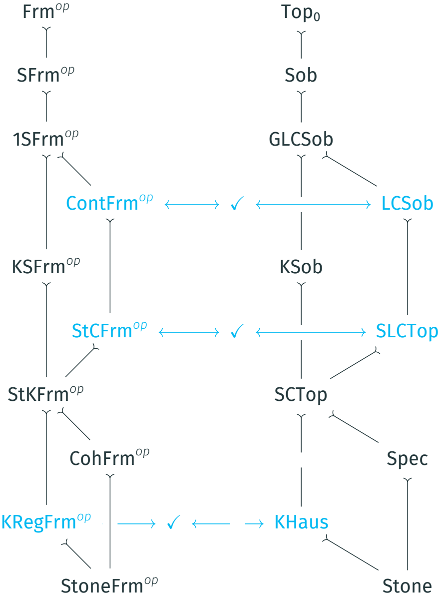

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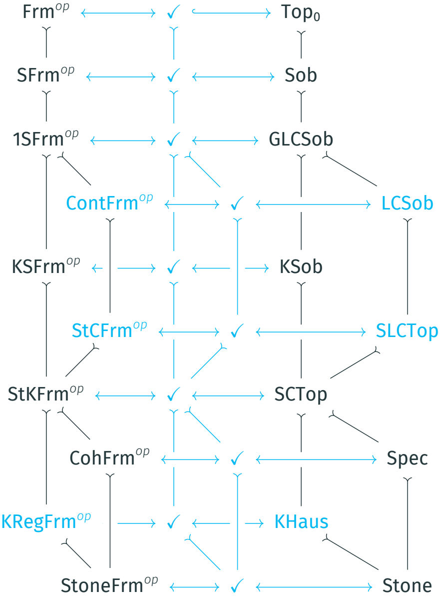

A new proof of:

- The Hofmann-Lawson duality between continuous frames and locally compact sober spaces
- The Johnstone duality between stably continuous frames and stably locally compact spaces.
- The Isbell duality between compact regular frames and compact Hausdorff spaces.
- Other (dual) equivalences.

Thank you!