Subordination Algebras as Semantic Environment of Input/Output Logic

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Input/output logic [11] has been introduced as a formal framework for modelling the interaction between logical inferences and other agency-related notions such as conditional obligations, goals, ideals, preferences, actions, and beliefs. This framework has been applied mainly in the context of the formalization of normative systems in philosophical logic and AI. Although, initially, this framework was intended "not [for] studying some kind of non-classical logic, but [as] a way of using the classical one", its generality and versatility makes it very suitable to support a range of enhancements in its expressiveness, such as those brought about by the addition of modal operators. Moreover, recently, there has been an interest in studying the interaction between the agency-related notions mentioned above with various forms of *nonclassical* reasoning [13, 14]. This interest has contextually motivated the introduction of algebraic and proof-theoretic methods in the study of input/output logic [15].

In this talk, we contribute to the latter research direction in the mathematical background of input/output logic (as defined in [11]) by introducing an algebraic semantics for it, based on (generalizations of) *subordination algebras* [1]. These can be defined as tuples $(A, \prec)$ such that A is a Boolean algebra and $\prec$ is a binary relation on A such that the direct (resp. inverse) image of each element $a \in A$ is a filter (resp. an ideal) of A. Subordination algebras are equivalent presentations of pre-contact algebras [10] and quasi-modal algebras [2, 3]. Since their introduction, subordination algebras have been systematically connected with various modal algebras (i.e. Boolean algebras expanded with semantic modal operators). This has made it possible to endow various modal languages with algebraic semantics based on subordination algebras, and use these languages to axiomatize the properties of these subordination algebras. In particular, Sahlqvist-type canonicity for modal and tense formulas on subordination algebras has been studied in [8] using topological techniques; in [9], using algebraic techniques, the canonicity result of [8] was strengthened and captured within the more general notion of canonicity in the context of *slanted algebras*, which was established using the tools of *unified correspondence theory* [5, 6, 7]. Slanted algebras are based on general lattices, and encompass variations and generalizations of subordination algebras such as those very recently introduced by Celani in [4], which are based on distributive lattices, and for which Celani develops duality-theoretic and correspondence-theoretic results.

In this talk, we propose a semantic framework in which the subordination relations $\prec$ of (proto-)subordination algebras interpret the normative system N of input/output logic. This makes it possible to conceptually interpret the meaning of $\prec$ in terms of the behaviour of systems of norms, to systematically relate rules of N with properties of $\prec$, and to interpret the output operators induced by N as the modal operators associated with $\prec$. We characterize a number of basic properties of N (or of $\prec$) in terms of modal axioms in this language. Interestingly, some of these properties are well known in the literature of input/output logic, since they capture intuitive desiderata about the interaction between norms and logical reasoning; some other properties originate from purely mathematical considerations, and have been dually characterized in Celani's correspondence-theoretic results in [4]. Thanks to the embedding of subordination algebras in the more general environment of slanted algebras, we have a mathematical environment for systematically exploring the interaction between norms and different modes of reasoning, and a systematic connection with families of

logical languages which can be applied in different contexts to axiomatize the behaviour of various generalized systems of norms; this more general environment allows to also encompass results such as Celani's correspondence for subordination lattices as consequences of *standard* Sahlqvist modal correspondence.

Similar to the duality between necessity and possibility in modal logic, the notion of conditional permission (sometimes referred to as negative permission) has been introduced as dual to conditional obligation: "a code permits something with respect to a condition iff it does not forbid it under that same condition, i.e. iff the code does not require the contrary under that condition" [12]. The duality between subordination relations and pre-contact relations [10] allows us to propose precontact relations for modelling conditional permission. Time permitting, we will discuss the relationship between pre-contact algebra and different concepts of permissions proposed by Makinson and van der Torre [12].

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