Interrogative agendas, categorization, and decision-making

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The ongoing work we wish to discuss is based on the preliminary results in [1], where we initiate a line of research aimed at formally modelling various types of decision-making processes in terms of categorization processes.

In this work, we explore the role of the epistemic stances of different agents (decision-makers) played in the decision-making processes. We model those epistemic stances as *interrogative agendas* [2], a notion introduced in epistemology and formal philosophy indicating the set of questions individual agents (or groups of agents) are interested in, or what they consider relevant for deciding, relative to a certain circumstance (independently of whether they utter the questions explicitly). Interrogative agendas might differ for the same agent in different moments or in different contexts; for instance, my interrogative agenda when I have to decide which car to buy will be different from my interrogative agenda when I listen to a politician's speech. Deliberation and negotiation processes can be understood in terms of whether and how decision-makers/negotiators succeed in modifying their own interrogative agendas or those of their counterparts, and the outcomes of these processes can be described in terms of the "common ground" agenda thus reached.

An influential approach in logic [4] represents questions as equivalence relations over a suitable set of possible worlds W. When ordered by inclusion, the set of equivalence relations on any set W is a complete (possibly non-distributive) lattice $E(W)$. Although the lattices $E(W)$ described above are in general not distributive, they resemble the powerset algebras in some important respects, for instance in their being completely join-generated and meet-generated by their atoms and co-atoms, respectively.

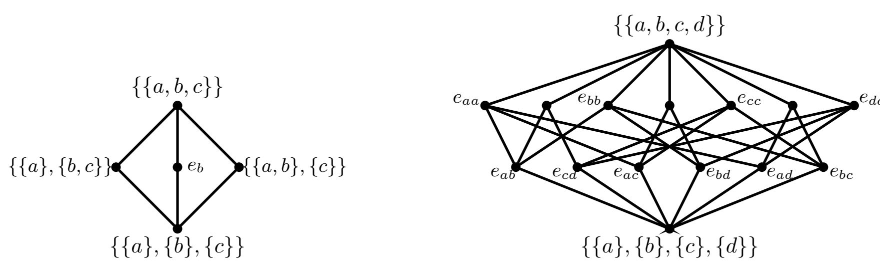

Figure 1: The lattices of equivalence relations on the three-element set $W := \{a, b, c\}$, and the four-element set $W := \{a, b, c, d\}$. In the lattice on the left, e_b corresponds to the partition $\{\{b\}, \{a, c\}\}$. In the lattice on the right, $e_{xy} = \{\{x\}, \{y\}, W \setminus \{x, y\}\}$ for all $x, y \in \{a, b, c, d\}$, and the unlabelled nodes correspond, from left to right, to the partitions $\{\{a, b\}, \{c, d\}\}$, $\{\{a, c\}, \{b, d\}\}$, and $\{\{a, d\}, \{b, c\}\}$, respectively.

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It is well known that every lattice is a sublattice of the lattice of all equivalence relations on some set [5]. This immediately implies that the lattice logic (or the basic non-distributive logic) is sound and complete w.r.t. the class of all lattices $E(W)$ described above. Hence, in the logical framework we will discuss, we propose that the basic non-distributive logic can be regarded as the basic logic of interrogative agendas. This basic framework naturally lends itself to be enriched with various kinds of logical operators, such as epistemic operators, which represent the way in which the interrogative agenda of an agent (or a group of agents) is perceived or known by another agent (or group), and dynamic operators, which encode the changes in agents' interrogative agendas. This basic framework can be further enriched with heterogeneous operators, suitable to encode the interaction among different kinds of entities; for instance, operators that associate (groups of) agents c with their (common) interrogative agenda $\Diamond c$, or operators that associate pairs (e, ϕ), such that e is an interrogative agenda and ϕ is a formula, with the formula $e \rhd \phi$, representing the content of ϕ 'filtered through' the interrogative agenda e. On the basis of these ideas, a fully-fledged formal epistemic theory of the interrogative agendas of social groups and individuals can be developed.

In this talk we will discuss a semantic framework, based on *formal contexts* [3], in which multiple agents are to categorize objects based on their own views of which features are relevant.

A *formal context* is a structure $\mathbb{P} = (A, X, I)$, representing a database of objects in the set A, features in the set X, and $I \subseteq A \times X$ recording which objects have which features.[1] The epistemic attitudes of the agents who are given the task of categorizing objects in A (and who might consider different subsets of X as relevant for their categorization task) are modelled by associating each agent with a different element of $E(X^*)$, where $X^* := X \cup \{x^*\}$ (see footnote below). In particular, if an agent considers the features in $Y \subseteq X$ as those of relevance, this agent is associated with the element $e_Y \in E(X^*)$ which is identified (meet-generated) by the meet irreducible elements of $E(X^*)$, corresponding to the bi-partitions $\{\{x\}, X^* \setminus \{x\}\}$ for every $x \in Y$. Partitions of the form e_Y, $Y \subseteq X$ form a sub-lattice of $E(X^*)$. We represent the categorization performed by an agent with interrogative agenda e_Y as above by the concept lattice of the formal context $\mathbb{P}_Y := (A, Y, I_Y)$ where $I_Y := I \cap (A \times Y)$.

The framework described above can be further enriched with additional relations giving rise to modal operators among different agents, agendas and categorizations, so to describe deliberation scenarios to model multi-agent interaction involving categorizations tasks such as auditing procedures.[2]

References

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[1] Each such $\mathbb{P}$ can be associated with $\mathbb{P}^* := (A, X^*, I^*)$, where $X^* := X \cup \{x^*\}$, and $I^* := I \cup \{(a, x^*) \mid a \in A\}$. Intuitively, x^* is a redundant feature, since it does not tell apart any object from any other. Clearly, $\mathbb{P}$ and $\mathbb{P}^*$ give rise to isomorphic concept lattices.

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